

INTEGRATION

If $F'(x) = f(x)$ for all x -interval then $F(x)$ is called integral or Anti derivative for all values of $f(x)$ for that interval.

$$\text{i.e. } \frac{d}{dx} F(x) = f(x)$$

$$d F(x) = f(x) dx.$$

$$\int d F(x) = \int f(x) dx.$$

$$\therefore \int f(x) dx = F(x) + c.$$

$$\int f(x) dx = F(x) + c$$

$\xrightarrow{\text{Integral}}$
 $\xrightarrow{\text{Constant of integral}}$
 $\xrightarrow{\text{operator.}}$
 $\xrightarrow{\text{Integrated function}}$
 $\xrightarrow{\text{sign of integration}}$

$$\int dx = x$$

dx — driver, $f(x)$ — power

$$\begin{aligned} dx &\rightarrow x \\ \int f(x) &\rightarrow x \\ f(x) &\rightarrow x \\ f(x) dx &\rightarrow x \end{aligned}$$

$$\begin{aligned} \int dx &= x \\ x^0 &= x \\ \frac{x^{0+1}}{1} &= x \\ x^1 &= x \\ \boxed{x} &= x \end{aligned}$$

TYPES OF INTEGRATION

Indefinite
(no limit)
 $\int f(x) dx = F(x) + c$

Definite.
(Having limit)
 $\int_a^b f(x) dx$
[a → lower limit
 b → upper limit]

Definite integral.

$$\int_a^b f(x) dx = [F(x)]_a^b$$

$F(b) - F(a)$ upper - lower.

Ex. $\int_2^3 f(4x) dx = [F(4x)]_2^3$

$$4(3) - 4(2) =$$
$$\Rightarrow 4$$

$$(i) \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Where $c \rightarrow$ constant of indefinite.

$$(ii) \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c.$$

$$\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1}$$

$$(11) \int \frac{1}{f(x)} \cdot f'(x) dx = \ln f(x) + c.$$

Evaluate the following ^{indefinite.} Integral

Q1. $\int (x^2 + 4x + 13) dx$

Sol $\int (x^2 + 4x + 13) dx$

$$\Rightarrow \int x^2 dx + \int 4x dx + \int 13 dx$$

$$\Rightarrow \left\{ \frac{x^{2+1}}{2+1} + 4 \int x dx + 13 \int dx \right.$$

$$\Rightarrow \frac{x^3}{3} + \frac{(4)x^2}{2} + 13x + c.$$

$$\Rightarrow \frac{x^3}{3} + 2x^2 + 13x + c \quad \underline{\text{Ans.}}$$

Q2 $\int (3x^4 - 5x^3 - 4x^2 - 2) dx$

Sol

$$\int (3x^4 - 5x^3 - 4x^2 - 2) dx$$

$$\Rightarrow 3 \int x^4 dx - 5 \int x^3 dx - 4 \int x^2 dx - 2 \int dx$$

$$\Rightarrow 3 \frac{x^5}{5} - 5 \frac{x^4}{4} - 4 \frac{x^3}{3} - 2x + c$$

Ans

Sol

$$\Rightarrow \int x^2 (x^2 - 4) dx$$

$$\Rightarrow \int (x^4 - 4x^2) dx$$

$$\Rightarrow \int x^4 dx - 4 \int x^2 dx$$

$$\Rightarrow \left\{ \frac{x^{4+1}}{5} - 4 \left(\frac{x^3}{3} \right) + C \right.$$

$$\Rightarrow \frac{x^5}{5} - \frac{4}{3} x^3 + C$$

Q5) $\int \sqrt{2x+5} dx \rightarrow (i)$

Let

$$2x+5 = u \rightarrow (ii)$$

Diff w.r.t (ii) w.r.t (x).

$$2 \frac{d(x)}{dx} + d5 = \frac{du}{dx}$$

$$\text{or } \frac{du}{dx} = 2$$

$$dx = \frac{1}{2} du$$

$$\Rightarrow \int \sqrt{2x+5} \frac{du}{dx} dx$$

$$\frac{1}{2} \int \sqrt{2x+5} (2) dx$$

$$\therefore \text{Equ (i)} \Rightarrow \int \sqrt{u} \left(\frac{1}{2} du \right)$$

$$\frac{1}{2} \cdot \frac{(2x+5)^{3/2}}{3/2} + C$$

$$\Rightarrow \frac{1}{2} \int (u)^{1/2} du$$

$$3(2x+5)^{3/2} + C$$

$$\Rightarrow \frac{1}{2} \times \frac{(u)^{3/2}}{3/2} + C$$

$$\Rightarrow \frac{1}{2} \times \frac{2}{3} (u)^{3/2} + C$$

$$\Rightarrow \frac{1}{3} (2x+5)^{3/2} + C$$

Ans

Q.7

$$Q4 \int e^{2-3x} dx = ?$$

(i) Q.7

$$\Rightarrow \int e^{2-3x} dx$$

$$\text{Put } 2-3x = u$$

$$-3dx = du$$

$$dx = -\frac{du}{3}$$

$$\Rightarrow -\frac{1}{3} \int e^u du$$

$$\Rightarrow -\frac{e^u}{3} + C$$

$$\Rightarrow -\frac{e^{2-3x}}{3} + C$$

$$Q2 \int \frac{x^2 dx}{e^{2x^3+3}}$$

$$\text{Let } 2x^3+3 = u$$

$$6x^2 dx = du$$

$$x^2 dx = \frac{du}{6}$$

2

$$\Rightarrow \frac{1}{6} \int \frac{1}{e^u} du$$

$$\Rightarrow \frac{1}{6} \int e^{-u} du$$

$$\Rightarrow \frac{1}{6} \frac{e^{-u}}{-1} + C$$

$$\Rightarrow -\frac{1}{6} e^{-u} + C$$

$$\Rightarrow -\frac{1}{6} e^{-(2x^3+3)} + C$$

$$\frac{d}{dx} a^x = a^x \ln a$$

Q $y = a^x$

$$= e^{\ln a^x}$$

$$= e^{x \ln a}$$

$$\frac{dy}{dx} = e^{x \ln a} \cdot \frac{d}{dx} x \ln a$$

$$= \cancel{e^{x \ln a}} \left[\cancel{x} \frac{d}{dx} \ln a + \ln a \frac{d}{dx} \cancel{x} \right]$$

$$\Rightarrow \cancel{e^{x \ln a}} x$$

$$= e^{x \ln a} \frac{d}{dx} (x \ln a)$$

$$= e^{x \ln a} \left[\ln a \frac{d}{dx} x \right]$$

$$= e^{x \ln a} \ln a$$

$$\frac{dy}{dx} = a^x \ln a$$

Q $\frac{dy}{dx} y = ?$

$$y = 5^x$$

$$y = 9^x$$

$$y = 6^x$$

$$y = 7^x$$

Q $\int a^{2y} dy$

$$\Rightarrow \int e^{\ln a^{2y}} dy$$

$$\Rightarrow \int e^{2y \ln a} dy$$

put

$$2y \ln a = u$$

$$2 \ln a \, dy = du$$

$$dy = \frac{du}{2 \ln a}$$

$$\Rightarrow \frac{1}{2 \ln a} \int e^u du$$

$$\Rightarrow \frac{1}{2 \ln a} e^u + C$$

$$\Rightarrow \frac{1}{2 \ln a} e^{2y \ln a} + C$$

$$\Rightarrow \frac{1}{2 \ln a} a^{2y} + C$$

Area

$$\oint \int a^{2y} dy = ?$$

Sol

$$\text{Let } a^{2y} = u$$

$$2 dy = du$$

$$\Rightarrow \frac{1}{2} \int a^u du$$

\Rightarrow Using

$$\boxed{\int a^x dx = \frac{a^x}{\ln a} + C}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{a^u}{\ln a} + C$$

$$\therefore \frac{1}{2} \cdot \frac{a^{2y}}{\ln a} + C$$

8.3

$$4 \text{ Q (iv) } \int \frac{e^{\frac{1}{u}}}{u^2} du = ?$$

$$\Rightarrow \int \frac{e^{\frac{1}{u}}}{u^2} du$$

$$\text{Let } \frac{1}{u} = t$$

$$\Rightarrow (-1) \frac{1}{u^2} du = dt$$

$$\Rightarrow -\frac{1}{u^2} du = dt$$

$$\therefore \int e^t (-dt)$$

$$\Rightarrow (-1) \int e^t dt$$

$$\Rightarrow (-1) e^t + c$$

$$\Rightarrow -e^{\frac{1}{u}} + c \quad \text{Ans}$$

$$\text{Q } \int e^x (2e^{3x} - 5)^{2/5} dx$$

$$(2e^{3x} - 5)^{2/5} \int e^x dx$$

$\therefore k = \text{constant}$

i.e. $(2e^{3x} - 5)^{2/5} = \text{constant}$

$$(2e^{3x} - 5)^{2/5} \int e^x dx$$

$$\boxed{(2e^{3x} - 5)^{2/5} e^x + c} \quad \text{Ans}$$

Chpt# 8.

①

A NTIDERIVATIVE.

Formula:-

$$\boxed{1} \int x^n dx = \frac{x^{n+1}}{n+1} + c, \forall n \neq -1$$

where, c = constant of integration.

$$Q1 // \int x^3 dx$$

$$\begin{aligned} &= \frac{x^{3+1}}{3+1} + c \\ &= \frac{x^4}{4} + c \end{aligned}$$

$$Q:- \int x^{-2} dx.$$

$$= \frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{x^{-1}}{-1} + c$$

$$= -\frac{1}{x} + c$$

$$Q:- \int \sqrt{x} dx$$

$$= \int (x)^{\frac{1}{2}} dx.$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{2x^{3/2}}{3} + c$$

$$Q:- \int x^3 dx.$$

$$= \frac{x^4}{4} + c$$

Let

$$y = \frac{x^4}{4} + c$$

diff w.r.t x,

$$\frac{dy}{dx} = \frac{4x^3}{4} + 0$$

$$\frac{dy}{dx} = x^3$$

$$\frac{dy}{dx} = x^3 dx$$

$$Q:- \int x^{4/5} dx$$

$$= \frac{x^{4/5+1}}{4/5+1} + c$$

$$= \frac{x^{9/5}}{9/5}$$

$$= \frac{5}{9} x^{9/5} + c$$

$$Q:- \int (x)^{-2/3} dx.$$

$$Q:- \int x^{-4} dx.$$

$$= -\frac{1}{3x^3} + c$$

$$= \frac{x^{-2/3+1}}{-2/3+1} + c$$

$$= \frac{x^{1/3}}{1/3}$$

$$= 3x^{1/3} + c$$

Note:-

$$\int dx = x + c$$

Proof:-

$$\int dx = \int 1 \cdot dx$$

$$= \int x^0 dx$$

$$= \frac{x^{0+1}}{0+1} + c$$

$$\int dx = x + c$$

$$Q:- \int (x)^{3/5} dx.$$

$$= \frac{x^{-3(5)+1}}{-3/5+1} + c$$

$$= \frac{5x^{2/5}}{2} + c$$

$$Q :- \int x^{7/8} dx = \frac{7x^{8/8}}{8} + c$$

Properties

$$\boxed{1} \int k \cdot x^n dx = k \int x^n dx.$$

where, $k = \text{constant}.$

$$\boxed{2} \int$$

$$\int [f(x) + g(x) + h(x) + u(x) + v(x)] dx = \int f(x) dx + \int g(x) dx + \int h(x) dx + \int u(x) dx + \int v(x) dx.$$

$$Q :- \int (x^3 + x^4 + x^2 + x^5 + 2x) dx$$

$$= \int x^3 dx + \int x^4 dx + \int x^2 dx + \int x^5 dx + 2 \int x dx.$$

$$\Rightarrow \frac{x^4}{4} + \frac{x^5}{5} + \frac{x^3}{3} + \frac{x^6}{6} + \frac{2x^2}{2} + c$$

$$\Rightarrow \boxed{\frac{1}{4}x^4 + \frac{1}{5}x^5 + \frac{1}{3}x^3 + \frac{1}{6}x^6 + x^2 + c}$$

Ans

5, 6, 10, 7

$$Q :- \int (x^{1/3} - 2x^{4/5} + x^{3/2} + \sqrt{x}) dx.$$

$$= \frac{x^{1/3+1}}{\frac{1}{3}+1} - 2 \frac{x^{4/5+1}}{\frac{4}{5}+1} + \frac{x^{3/2+1}}{\frac{3}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{x^{4/3}}{4/3} - 2 \frac{x^{9/5}}{9/5} + \frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + C$$

$$= \left[\frac{3}{4} x^{4/3} - \frac{10}{9} x^{9/5} + \frac{2}{5} x^{5/2} + \frac{2}{3} x^{3/2} + C \right]$$

8.1

$$Q 1 \int (x^2 + 4x + 13) dx$$

Sol

$$= \int (x^2 + 4x + 13) dx$$

$$= \int x^2 dx + 4 \int x dx + 13 \int dx.$$

$$= \frac{1}{3} x^3 + \frac{4}{2} x^2 + 13x + C$$

$$= \frac{1}{3} x^3 + 2x^2 + 13x + C \quad \underline{\text{Ans}}$$

$$Q 11 // \int x (x^3 + 1)^2 dx.$$

(2)

$$= \int x(x^3 + 1)^4 dx.$$

$$= \int x(x^6 + 2x^3 + 1) dx$$

$$= \int (x^7 + 2x^4 + x) dx$$

$$\Rightarrow \int x^7 dx + 2 \int x^4 dx + \int x dx$$

$$= \frac{x^8}{8} + 2 \cdot \frac{x^5}{5} + \frac{x^2}{2} + C$$

$$= \frac{1}{8} x^8 + \frac{2}{5} x^5 + \frac{1}{2} x^2 + C$$

$$Q2 \int (3x^4 - 5x^3 - 4x^2 - 2) dx.$$

$$\Rightarrow \int (3x^4 - 5x^3 - 4x^2 - 2) dx$$

$$\Rightarrow 3 \int x^4 dx - 5 \int x^3 dx - 4 \int x^2 dx - 2 \int dx.$$

$$\Rightarrow \frac{3x^5}{5} + \frac{-5x^4}{4} - \frac{4x^3}{3} - 2x + C$$

$$\Rightarrow \frac{3}{5} x^5 - \frac{5}{4} x^4 - \frac{4}{3} x^3 - 2x + C$$



$$Q4 \int x^2(x^2 - 4) dx$$

$$\Rightarrow \int x^2(x^2 - 4) dx$$

$$\Rightarrow \int x^4 - 4x^2 dx$$

$$\Rightarrow \int x^4 - 4 \int x^2 dx$$

$$\Rightarrow \frac{x^5}{5} - \frac{4x^3}{3} + C \quad \underline{\underline{\text{Ans}}}$$

$$\text{Q8 } \int \frac{dx}{u^2}$$

Sol

$$\Rightarrow \int \frac{du}{u^2}$$

$$\Rightarrow \int u^{-2} du$$

On integrating

$$\Rightarrow \frac{u^{-2+1}}{-2+1} + C$$

$$\Rightarrow -\frac{u^{-1}}{1} + C$$

$$\Rightarrow -\frac{1}{u} + C \quad \underline{\underline{\text{Ans}}}$$

$$\text{Q10 } \int \frac{du}{\sqrt{a}}$$

$$\text{Q11 } \int x(x^3+1)^2 dx$$

Sol

$$\Rightarrow \int x \{x^3+1\}^2 dx$$

$$\Rightarrow \int x \{ (x^3)^2 + 2(x^3)(1) + (1)^2 \} dx$$

$$\Rightarrow \int x \{ x^6 + 2x^3 + 1 \} dx$$

$$\Rightarrow \int (x^7 + 2x^4 + x) dx$$

$$\text{Q9 } \int \frac{6}{u^5} du \Rightarrow ?$$

Sol

$$\Rightarrow \int \frac{6}{u^5} du$$

$$\Rightarrow \int x^7 dx + 2 \int x^4 dx + \int x dx$$

$$\Rightarrow \frac{x^8}{8} + 2 \frac{x^5}{5} + \frac{x^2}{2} + C$$

$$\Rightarrow 6 \int u^{-5} du = \frac{1}{8} x^8 + \frac{2}{5} x^5 + x^2 \cdot \frac{1}{2} + C$$

$$\Rightarrow 6 \left[\frac{u^{-5+1}}{-5+1} \right] + C$$

Ans

$$\Rightarrow 6 \left(\frac{u^{-4}}{-4} \right) + C$$

$$= -\frac{3}{2} u^{-4} + C \quad \underline{\underline{\text{Ans}}}$$

Exam pg 215

$$\int (ax^3 + bx^2 + cx + e) dx = ?$$

Sol

$$\int (ax^3 + bx^2 + cx + e) dx$$

$$\Rightarrow a \int x^3 dx + b \int x^2 dx + c \int x dx + e \int dx.$$

$$\Rightarrow \frac{a}{4} x^4 + \frac{b}{3} x^3 + \frac{c}{2} x^2 + ex + C.$$

A

FORMULAE.

QNN

$$\boxed{2} \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

where $n \neq -1$
C = const

A

Q

$$\int (3x+5)^{1/2} dx.$$

$$\Rightarrow \frac{(3x+5)^{1/2+1}}{3(\frac{1}{2}+1)} + C$$

$$\Rightarrow \frac{3^{1/3} (3x+5)^{4/3}}{3(\frac{4}{3})} + C$$

$$\Rightarrow \frac{(3x+5)^{4/3}}{4} + C$$

Q

$$\int \sqrt{6x-5} dx.$$

$$\rightarrow \int (6x-5)^{1/2} dx.$$

$$\Rightarrow \frac{(6x-5)^{1/2+1}}{6(\frac{1}{2}+1)} + C$$

$$\Rightarrow \frac{(6x-5)^{3/2}}{\left(\frac{3}{2}\right) \times 3} + C$$

$$\Rightarrow \boxed{\frac{(6x-5)^{3/2}}{9} + C} \quad \underline{\text{Ans}} \quad 6, 7, 10, 18$$

Q15 //

$$\int \sqrt{2x+5} \, dx$$

sl

$$\Rightarrow \int (2x+5)^{1/2} \, dx$$

$$\therefore \int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\Rightarrow \frac{(2x+5)^{1/2+1}}{2\left(1+\frac{1}{2}\right)} + C$$

$$\Rightarrow \boxed{\frac{(2x+5)^{3/2}}{3} + C} \quad \underline{\text{Ans}}$$

$$Q13 \int (\sqrt{x}-1)^2 \, dx$$

$$\Rightarrow \int (x - 2\sqrt{x} + 1) \, dx$$

$$\Rightarrow \int x \, dx - 2 \int x^{1/2} \, dx + \int 1 \, dx$$

$$\Rightarrow \frac{1}{2} x^2 - 2 \frac{x^{3/2}}{3/2} + x + C$$

$$\Rightarrow \frac{1}{2} x^2 - \frac{4}{3} x^{3/2} + x + C \quad \underline{\text{Ans}}$$

$$Q17 // \int \frac{(\sqrt{\theta}-1)^3}{\sqrt{\theta}} \, d\theta \quad 15, 16.$$

$$\Rightarrow \int \left[\frac{(\sqrt{\theta})^3 - (1)^3 - 3(\sqrt{\theta})(1)(\sqrt{\theta}-1)}{\sqrt{\theta}} \right] d\theta$$

(3)

$$\Rightarrow \int \left[\frac{\theta^{3/2} - 1 - 3\theta^{1/2}(\theta^{1/2} - 1)}{\theta^{1/2}} \right] d\theta$$

$$\Rightarrow \int \left[\frac{\theta^{3/2} - 1 - 3\theta + 3\theta^{1/2}}{\theta^{1/2}} \right] d\theta$$

$$\Rightarrow \int \left[\frac{\theta^{3/2}}{\theta^{1/2}} - \frac{1}{\theta^{1/2}} - \frac{3\theta}{\theta^{1/2}} + 3 \frac{\theta^{1/2}}{\theta^{1/2}} \right] d\theta$$

$$\Rightarrow \int (\theta - \theta^{-1/2} - 3\theta^{1/2} + 3) d\theta$$

$$\Rightarrow \int \theta d\theta - \int \theta^{-1/2} d\theta - 3 \int \theta^{1/2} d\theta + 3 \int d\theta$$

$$\Rightarrow \frac{\theta^2}{2} - \frac{\theta^{1/2}}{(-1/2)} - 3 \frac{\theta^{3/2}}{(3/2)} + 3\theta + c.$$

$$\Rightarrow \frac{\theta^2}{2} - 2\theta^{1/2} - \frac{6}{3} \theta^{3/2} + 3\theta + c.$$

FORMULA

$$\int f^n(x) f'(x) dx = \frac{f^{n+1}(x)}{n+1} + c.$$

[For. Equivalent to Algebraic]
substitution

$\forall n \neq -1$

$$Q \int (x^3+1)^5 (3x^2) dx,$$

$$\Rightarrow \frac{(x^3+1)^6}{6} + C \quad \therefore$$

$$Q :- \int (\sqrt{x^2-1})^{0.2} dx$$

$$\Rightarrow \int (x^2-1)^{1/2} (2x) dx$$

$$\Rightarrow \frac{2}{3} (x^2-1)^{3/2} + C$$

$$Q :- \int (x^4+6)^{3/2} \frac{4}{4} x^3 dx$$

$$\Rightarrow \frac{1}{4} \int (x^4+6)^{3/2} 4x^3 dx$$

$$\Rightarrow \frac{1}{4} \left[\frac{(x^4+6)^{5/2}}{5/2} \right] + C$$

$$\Rightarrow \frac{1}{10} (x^4+6)^{5/2} + C.$$

$$\Rightarrow \frac{1}{10} (x^4+6)^{5/2} + C$$

$$Q 11 \int \frac{(3x^2+2x+1) dx}{(x^3+x^2+x+7)^{1/7}}$$

$$\Rightarrow \int (x^3+x^2+x+7)^{-1/7} (3x^2+2x+1) dx.$$

$$\Rightarrow \frac{7}{6} (x^3+x^2+x+7)^{6/7} + C$$

$$\Rightarrow \frac{7}{6} (x^3+x^2+x+7)^{6/7} + C$$

8.3

Q1, $\int 4x^3 (x^4+1)^{5/2} dx$

$$\int (x^4+1)^{5/2} 4x^3 dx$$

$$\Rightarrow \frac{(x^4+1)^{7/2}}{7/2} + C$$

$$= \frac{2}{7} (x^4+1)^{7/2} + C$$

8.3

Q 1 (ii) $\int \frac{8x^2}{(x^2+2)^3} dx$

$$\Rightarrow \int (x^2+2)^{-3} (8x^2) dx$$

$$\Rightarrow \int (x^2+2)^{-3} \left(\frac{8}{3} \times 3x^2 \right) dx$$

$$\Rightarrow \frac{8}{3} \int (x^2+2)^{-3} \cdot 3x^2 dx$$

$$\Rightarrow \frac{8}{3} \left[\frac{(x^2+2)^{-2}}{-2} \right] + C$$

$$\Rightarrow -\frac{8}{6} (x^2+2)^{-2} + C$$

$$\Rightarrow -\frac{4}{3} (x^2+2)^{-2} + C$$

Q1(ii)³ $\int \sqrt{1-2x^2} (3x) dx$

$$\Rightarrow \int (1-2x^2)^{1/2} (3x) dx$$

$$\Rightarrow \int \left[\frac{(1-2x^4)^{1/2+1}}{3/2} \right] + C$$

$$= \left[\frac{2(1-2x^4)^{3/2}}{3} + C \right] \quad \underline{\text{Ans}}$$

Q 1 (iv) $\int \frac{y+3}{(y^2+6y)^{1/2}} dy$

$$\Rightarrow \int (y^2+6y)^{-1/2} (y+3) dy$$

$$\Rightarrow \int (y^2+6y)^{-1/2} \frac{1}{2} (2y+6) dy$$

$$\Rightarrow \frac{1}{2} \int (y^2+6y)^{-1/2} (2y+6) dy$$

$$\Rightarrow \frac{1}{2} \left[\frac{(y^2+6y)^{1/2}}{1/2} \right] + C$$

$$\Rightarrow \left[(y^2+6y)^{1/2} \right] + C \quad \underline{\text{Ans}}$$

Q 1 (viii) $\int \frac{x^2 dx}{(1-2x^3)^{2/3}}$

$$\Rightarrow \int (1-2x^3)^{-2/3} x^2 dx$$

$$\Rightarrow \int (1-2x^3)^{-2/3} \left(\frac{-6}{-6} \right) x^2 dx$$

$$\Rightarrow -\frac{1}{6} \int (1-2x^3)^{-2/3} (-6x^2) dx$$

$$\Rightarrow -\frac{1}{6} \left[\frac{(1-2x^3)^{1/3}}{1/3} \right] + C$$

$$\Rightarrow \left[-\frac{1}{2} (1-2x^3)^{1/3} + C \right] \quad \underline{\text{Ans}}$$

(4)

$$Q6 \int \sqrt{x^2 - 24x + 1} dx$$

$$\Rightarrow \int \sqrt{x^2(1 - 24x^2)} dx$$

$$\Rightarrow \int (1 - 24x^2)^{1/2} x dx$$

$$\Rightarrow \int (1 - 24x^2)^{1/2} - 48(x) dx$$

$$\Rightarrow \frac{1}{48} \int (1 - 24x^2)^{1/2} (-48x) dx$$

$$\Rightarrow \frac{(1 - 24x^2)^{3/2}}{3/2} + C$$

$$\Rightarrow \frac{2}{3} (1 - 24x^2)^{3/2} + C$$

Example #1 [2.24 page]

$$Q \int \frac{x dx}{\sqrt{4+x^2}} = ?$$

$$\Rightarrow \int (4+x^2)^{1/2} x dx$$

$$\Rightarrow \int (4+x^2)^{1/2} \frac{2x}{2} dx$$

$$\Rightarrow \frac{1}{2} \int (4+x^2)^{1/2} 2x dx$$

$$\Rightarrow \frac{1}{2} \left[\frac{(4+x^2)^{3/2}}{3/2} \right] + C$$

$$\Rightarrow \left[\frac{1}{3} (4+x^2)^{3/2} + C \right] \quad \text{Ans}$$

Q 6 ^{8.1}
 $\int (2x+3)^{2/3} dx$

18.1
 For 2

$\Rightarrow \int (2x+3)^{2/3} dx$
 $\Rightarrow \frac{1}{2} \int 2 \cdot (2x+3)^{2/3} dx$
 $= \frac{3}{5} (2x+3)^{5/3} + C$ Ans

Q 7 $\int (3x+4)^{29} dx$

$\Rightarrow \frac{1}{3} \int 3 (3x+4)^{29} dx$
 $\Rightarrow \frac{1}{3} \left[\frac{(3x+4)^{30}}{30} \right] + C$
 $\Rightarrow \frac{1}{90} (3x+4)^{30} + C$

Q 10 $\int \frac{dy}{\sqrt{ay+b}}$

$\Rightarrow \int (ay+b)^{-1/2} dy$
 $\Rightarrow \frac{1}{a} \int a (ay+b)^{-1/2} dy$
 $\Rightarrow \frac{1}{a} \frac{(ay+b)^{1/2}}{1/2} + C$
 $\Rightarrow \frac{2}{a} (ay+b)^{1/2} + C$ Ans

Q 18 $\int \frac{dx}{(2x+3)^{2/3}} = ?$

$\Rightarrow \int (2x+3)^{-2/3} dx$

$$\Rightarrow \frac{1}{2} \int (2x+3)^{-2/3} (2) dx$$

8.1

For: 2

$$\Rightarrow \frac{1}{2} \left[\frac{(2x+3)^{1/3}}{1/3} \right] + C$$

$$\Rightarrow \boxed{\frac{3}{2} [(2x+3)^{1/3}] + C} \quad \underline{\underline{A_1}}$$

Q 15 $\int \frac{x^2 + 3x^{1/2} + 4}{3x^{3/4}} dx$

Sol

$$\Rightarrow \frac{1}{3} \int \left(\frac{x^2}{x^{3/4}} + \frac{3x^{1/2}}{x^{3/4}} + \frac{4x^{-3/4}}{1} \right) dx$$

$$\Rightarrow \frac{1}{3} \int \left(x^{5/4} + 3x^{-1/4} + 4x^{-3/4} \right) dx$$

$$\Rightarrow \frac{1}{3} \int \left[\frac{x^{9/4}}{9/4} + \frac{3x^{3/4}}{3/4} + 4 \frac{x^{1/4}}{1/4} \right] + C$$

$$\Rightarrow \frac{1}{3} \left[\frac{4}{9} x^{9/4} + 4x^{3/4} + 16x^{1/4} \right] + C$$

A₂

Q 16 $\int \frac{(x+8)}{\sqrt{x}} dx = ?$

Sol

$$\int \left(\frac{x+8}{\sqrt{x}} \right) dx$$

$$\Rightarrow \int \left[\frac{x}{x^{1/2}} + 8 \frac{1}{x^{1/2}} \right] dx$$

$$\Rightarrow \int \left(x^{1/2} + 8x^{-1/2} \right) dx$$

$$\Rightarrow \int x^{1/2} dx + 8 \int x^{-1/2} dx$$

$$\Rightarrow \frac{x^{3/2}}{3/2} + 8 \frac{x^{1/2}}{1/2} + C$$

$$\Rightarrow \frac{2}{3} x^{3/2} + 16 x^{1/2} + C.$$

$$\Rightarrow \boxed{\frac{2}{3} x^{3/2} + 16 x^{1/2} + C.} \underline{Ans}$$

Q $\int \frac{(x^2 + 2x + 4)^{1/3}}{(x+1)} dx$

Sol

$$\Rightarrow \int \frac{(x+1)}{(x^2 + 2x + 4)^{-1/3}} dx$$

$$\Rightarrow \int (x^2 + 2x + 4)^{1/3} (x+1) dx$$

$$\Rightarrow \int (x^2 + 2x + 4)^{1/3} (x+1) dx$$

$$\Rightarrow \frac{1}{2} \int (x^2 + 2x + 4)^{1/3} (2(x+1)) dx$$

$$\Rightarrow \frac{1}{2} \left[\frac{(x^2 + 2x + 4)^{4/3}}{4/3} \right] + C$$

$$\Rightarrow \boxed{\frac{3}{8} (x^2 + 2x + 4)^{4/3} + C.} \underline{Ans}$$

Integral

Definite
(With limit)

Indefinite
(without limit)

$$\int_a^b f(x) dx = \text{upper limit} - \text{lower limit}$$

$b \rightarrow$ upper limit
 $a \rightarrow$ lower limit
 $= f(b) - f(a)$

Q

$$\int_1^2 2x dx$$

Sol

$$\Rightarrow 2 \int_1^2 x dx$$

$$\Rightarrow 2 \left[\frac{x^2}{2} \right]_1^2$$

$$\Rightarrow \left[x^2 \right]_1^2$$

$$\Rightarrow (2)^2 - (1)^2$$

$$\Rightarrow 4 - 1$$

$$\Rightarrow 3 \text{ Ans}$$

8.1

19

$$\int_{-1}^1 (2x^2+4)^3 (4x) dx$$

sol

$$\Rightarrow \frac{1}{4} \int_{-1}^1 \left[\frac{(2x^2+4)^4}{4} \right]_{-1}^1$$

$$\Rightarrow \frac{1}{4} \left\{ (2(1)^2+4)^4 - (2(-1)^2+4)^4 \right\}$$

$$\Rightarrow \frac{1}{4} \left\{ (2+4)^4 - (2+4)^4 \right\}$$

$$\Rightarrow \frac{1}{4} \left\{ (6)^4 - (6)^4 \right\}$$

$$\Rightarrow 0 \quad \underline{\text{Ans.}}$$

20

$$\int_0^2 (x^2+bx+c)^{2/3} \left(x+\frac{b}{2}\right) dx$$

sol

$$\Rightarrow \int_0^2 (x^2+bx+c)^{2/3} \left(\frac{2x+b}{2}\right) dx$$

$$\Rightarrow \frac{1}{2} \int_0^2 (x^2+bx+c)^{2/3} (2x+b) dx$$

$$\Rightarrow \frac{1}{2} \left[\frac{(x^2+bx+c)^{5/3}}{5/3} \right]_0^2$$

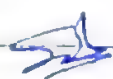
$$\Rightarrow \frac{3}{2} \left[(x^2+bx+c)^{5/3} \right]_0^2$$

$$\Rightarrow \frac{3}{2} \left[\{ (2)^2 + b \cdot 2 + c \}^{\frac{1}{3}} - \{ 0^2 + 0 \cdot b + c \}^{\frac{1}{3}} \right]$$

$$\Rightarrow \boxed{\frac{3}{2} \left[\{ 4 + 2b + c \}^{\frac{1}{3}} - c^{\frac{1}{3}} \right]} \quad \underline{\text{Ans}}$$

Q (14)

$$\int_0^2 \frac{dx}{\sqrt{1+x} + \sqrt{x}}$$



$$\Rightarrow \int_0^2 \left[\frac{dx}{\sqrt{1+x} + \sqrt{x}} \times \frac{\sqrt{1+x} - \sqrt{x}}{\sqrt{1+x} - \sqrt{x}} \right] dx$$

$$\Rightarrow \int_0^2 \frac{(\sqrt{1+x} - \sqrt{x})}{(\sqrt{1+x})^2 - (\sqrt{x})^2} dx$$

$$\Rightarrow \int_0^2 \frac{(\sqrt{1+x} - \sqrt{x})}{1+x-x} dx$$

$$\Rightarrow \int_0^2 (\sqrt{1+x} - \sqrt{x}) dx$$

$$\Rightarrow \int_0^2 \sqrt{1+x} dx - \int_0^2 \sqrt{x} dx$$

$$\Rightarrow \frac{2}{3} \left[(1+x)^{\frac{3}{2}} \right]_0^2 - \frac{2}{3} \left[x^{\frac{3}{2}} \right]_0^2$$

$$\Rightarrow \frac{2}{3} \left\{ (1+2)^{\frac{3}{2}} - (1+0)^{\frac{3}{2}} \right\} - \frac{2}{3} \left\{ 2^{\frac{3}{2}} - 0^{\frac{3}{2}} \right\}$$

$$3^1 \cdot 3^{1/2} \Rightarrow 3^{3/2}$$

$$814 \rightarrow 1 \rightarrow 2$$

$$7, 9, 13$$

$$\Rightarrow \frac{2}{3} \{ 3^{3/2} - 1^{3/2} \} - \frac{2}{3} \{ 2^{3/2} \}$$

$$\Rightarrow \frac{2}{3} \cdot 3^{3/2} - \frac{2}{3} - \frac{2^{3/2}}{3}$$

$$\Rightarrow 2 \cdot 3^{3/2-1} - \frac{2}{3} - \frac{2^{5/2}}{3}$$

$$\Rightarrow 2 \cdot 3^{1/2} - \frac{2}{3} - \frac{2^{5/2}}{3}$$

$$\Rightarrow \frac{2}{3} [3^{3/2} - 1^{3/2} - 2^{3/2}]$$

$$\Rightarrow \frac{2}{3} [3\sqrt{3} - 1 - 2\sqrt{2}]$$

B.4

8.3

$$\int_{-3}^{-1} \frac{dx}{(x-1)^3}$$

8.4

8.3

$$\Rightarrow \int_{-3}^{-1} (x-1)^{-3} dx$$

$$\Rightarrow \left[\frac{(x-1)^{-2}}{-2} \right]_{-3}^{-1}$$

$$\Rightarrow -\frac{1}{2} \left[(x-1)^{-2} \right]_{-3}^{-1}$$

$$\Rightarrow -\frac{1}{2} \{ (-1-1)^{-2} - (-3-1)^{-2} \}$$

$$\Rightarrow -\frac{1}{2} \{ (-2)^{-2} - (-4)^{-2} \} = -\frac{1}{2} \left[\frac{3}{32} \right]$$

Q.3

Algebraic substitution

(6)

$$Q1(i) \int 4x^3 (x^4 + 1)^{1/2} dx$$

Sol

Put

$$x^4 + 1 = t$$

$$4x^3 dx = dt$$

$$\Rightarrow \int (t)^{3/2} dt$$

$$\Rightarrow \left[\frac{t^{5/2}}{5/2} \right] + C$$

$$\Rightarrow \frac{2}{5} t^{5/2} + C$$

$$\Rightarrow \frac{2}{5} (x^4 + 1)^{5/2} + C$$

$$Q1:- \int \frac{x}{\sqrt{1+x}} dx$$

Sol

$$\Rightarrow \int (1+x)^{1/2} x dx$$

$$\text{Put } 1+x = t \quad \text{or } x = t-1$$

$$dx = dt$$

$$\Rightarrow \int (t)^{-1/2} (t-1) dt$$

$$\Rightarrow \int (t^{1/2} - t^{-1/2}) dt$$

$$\Rightarrow \frac{t^{3/2}}{3/2} - \frac{t^{1/2}}{1/2} + C$$

$$\Rightarrow \frac{2}{3} t^{3/2} - 2 t^{1/2} + C$$

$$\Rightarrow \frac{2}{3} (1+x)^{3/2} - 2 (1+x)^{1/2} + C$$

Q 1 (xiii) $\int (3x+2) (x-1)^{-1/2} dx$

$$\Rightarrow \int (x-1)^{-1/2} (3x+2) dx$$

$$\Rightarrow \text{put } x-1 = t \Rightarrow x = t+1 \\ dx = dt$$

$$\Rightarrow \int (t)^{-1/2} (3(t+1)+2) dt$$

$$\Rightarrow \int (t)^{-1/2} (3t+3+2) dt$$

$$\Rightarrow \int (t)^{-1/2} (3t+5) dt$$

$$\Rightarrow \int (3t^{1/2} + 5t^{-1/2}) dt$$

$$\Rightarrow 3 \int t^{1/2} dt + 5 \int t^{-1/2} dt$$

$$\Rightarrow 3 \left[\frac{t^{3/2}}{3/2} \right] + 5 \left[\frac{t^{1/2}}{1/2} \right] + C$$

$$\Rightarrow \frac{2}{3} 3 t^{3/2} + 10 t^{1/2} + C$$

$$\Rightarrow 2(x-1)^{3/2} + 10(x-1)^{5/2} + C$$

8 I (xii) $\int x \sqrt{4+x} dx$

$$\Rightarrow \int \sqrt{4+x} x^2 dx$$

put $4+x=t \Rightarrow x=t-4$
 $dt = dx$

$$\Rightarrow \int (t)^{1/2} \cdot (t-4)^2 dt$$

$$\Rightarrow \int t^{1/2} \cdot (t^2 - 8t + 16) dt$$

$$\Rightarrow \int (t^{5/2} - 8t^{3/2} + 16t^{1/2}) dt$$

$$\Rightarrow \int t^{5/2} dt - 8 \int t^{3/2} dt + 16 \int t^{1/2} dt$$

$$\Rightarrow \frac{2}{7} t^{7/2} - 8 \frac{t^{5/2}}{5/2} + 16 \frac{t^{3/2}}{3/2} + C$$

$$\Rightarrow \frac{2}{7} (4+x)^{7/2} - \frac{16}{5} (x+4)^{5/2} + \frac{32}{3} (4+x)^{3/2} + C$$

8 I (xiv) $\int (2x^2-3)^{1/3} x^3 dx$

$$\Rightarrow \int (2x^2-3)^{1/3} x^2 \cdot x dx$$

$$\Rightarrow \int (2x^2-3)^{1/3} x^2 \cdot x dx$$

$$t \Rightarrow x = \frac{t-3}{2}$$

$$dt = 2 dx$$

$$dx = \frac{dt}{2}$$

$$\Rightarrow \int (2x^2-3)^{1/3} x^2 \cdot \frac{dt}{2}$$

$$\frac{dy}{dx} = \ln \sin x$$

$$\frac{dy}{dx} = \frac{1}{\sin x} \frac{d(\sin x)}{dx} = \frac{\cos x}{\sin x} = \cot x$$

16) $\int (x^2 - 2x + 1)^{1/3} dx$

$$\Rightarrow \int [(x-1)^2]^{1/3} dx$$

$$\Rightarrow \int (x-1)^{2/3} dx$$

$$\Rightarrow \int \left[\frac{(x-1)^{5/3}}{5/3} \right] + C$$

$$\Rightarrow \frac{3}{5} \cdot (x-1)^{5/3} + C$$

Form $\frac{1}{x}$

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$$

Example:

① $\int \frac{1}{x} dx = \ln x + C$

② $\int \frac{\cos x}{\sin x} dx = \ln(\sin x) + C$

S.B.

Q1 (VII) $\int \frac{3x}{3x^2 - 4} dx$

Q9, 10.

Ans

$$\Rightarrow \frac{1}{6} \int \frac{6x}{3x^2 - 4} dx$$

$$\ln a^x = x \ln a$$

$$\Rightarrow \frac{1}{6} \ln(3x^2 - 4) + C$$

$$\Rightarrow \ln(3x^2 - 4)^{1/6} + C$$

1 =

$$8.3 = 10.9$$

$$8.4 = 1.7$$

own

$$\int \frac{2x+3}{x^2+3x} dx$$

$$\Rightarrow \ln(x^2+3x) + C$$

own

Q

$$\int \frac{x+1}{x^2+2x} dx$$

Q

$$= \frac{1}{2} \int \frac{2x+2}{x^2+2x} dx$$

$$\Rightarrow \frac{1}{2} \ln(x^2+2x) + C$$

$$\therefore \frac{f'(x)}{f(x)} dx = \ln f(x)$$

$$\Rightarrow \ln(x^2+2x)^{1/2} + C$$

$$\Rightarrow \ln \sqrt{x^2+2x} + C$$

8.3

Q 1 (iv)

$$\int \frac{x+2}{x-3} dx = ?$$

sk

$$\Rightarrow \int \frac{x+2}{x-3} dx$$

$$x-3 \overline{) x+}$$

$$Q + \frac{R}{D}$$

Quotient + Remainder
divisor.

$$\begin{array}{r} 2 \\ 2 \overline{) 5} \\ \underline{4} \\ 1 \end{array}$$

$$2 + \frac{1}{2}$$

$$\begin{array}{r} 1 \\ x-3 \overline{) x+2} \\ \underline{-(x-3)} \\ 5 \end{array}$$

$$\Rightarrow \int \left(1 + \frac{5}{x-3} \right) dx$$

$$\Rightarrow \int dx + 5 \int \frac{1}{x-3} dx$$

$$\Rightarrow x + 5 \ln(x-3) + C$$

$$\Rightarrow x + \ln(x-3)^5 + C$$

$$Q \int \frac{x+7}{x-3} dx$$

$$\begin{array}{r} 1 \\ x-3 \overline{) x+7} \\ \underline{-(x-3)} \\ 10 \end{array}$$

$$\Rightarrow \int \left(1 + \frac{10}{x-3} \right) dx$$

$$\Rightarrow \int dx + 10 \int \frac{1}{x-3} dx$$

$$\Rightarrow x + 10 \ln(x-3) + C$$

$$\Rightarrow x + \ln(x-3)^{10} + C$$

8.3
1 → 16
complete 5%

$$8 \int \frac{u+2}{u-3} du$$

Sub

$$\Rightarrow \int \frac{u+2}{u-3} du$$

$$u-3 \overline{) \begin{array}{r} u+2 \\ \underline{u-3} \\ 5 \end{array}}$$

$$\Rightarrow \int \left(1 + \frac{5}{u-3} \right) du$$

$$\Rightarrow \int du + 5 \int \frac{1}{u-3} du$$

$$\Rightarrow u + 5 \ln(u-3) + C$$

$$\Rightarrow u + \ln(u-3)^5 + C \quad \underline{\underline{Ans}}$$

$$\sin x + C$$

$$-\cos x + C$$

$$\tan x + C$$

$$\sec x + C$$

$$\ln \sin x$$

$$-\cot x + C$$

$$-\operatorname{cosec} x + C$$

$$\int \cos x dx$$

$$\int \sin x dx$$

$$\int \sec^2 x dx$$

$$\int \sec x \tan x dx$$

$$\int \cot x dx$$

$$\int \operatorname{cosec}^2 x dx$$

$$\int \operatorname{cosec} x \cot x dx$$

$$\ln x + c \quad \bigg| \quad \int \frac{1}{x}$$

(Q.3) Q 3 (vi).

$$\int \tan x \, dx$$

$$(\cos x)^{-1} \neq \cos^{-1}$$

$$\Rightarrow \int \frac{\sin x}{\cos x} \, dx$$

↓
Inverse
trig
metric
function

$$\Rightarrow - \int \frac{-\sin x}{\cos x} \, dx$$

$$\Rightarrow (-1) \ln (\cos x) + c$$

$$\Rightarrow \ln (\cos x)^{-1} + c$$

$$\Rightarrow \ln \left(\frac{1}{\cos x} \right) + c$$

$$\Rightarrow \ln \sec x + c$$

Note:-

trigonometric Single Function
فردی تریگونومیکی فونکشن
- فرض کریں θ Suppose Angle (θ)

$$\int \sin (3x+2) \, dx$$

$$\text{let } u = 3x+2$$

or Suppose Tri. Fun $\sin u$ (فردی تریگونومیکی فونکشن)
let u be the Derivative of x (فردی تریگونومیکی فونکشن کی ڈی ریویٹیو)

$$\int \sin x \cdot \cos x \, dx$$

$$\text{Let } \sin x = u$$

(8)

Proof:- $\int \sin(3x+2) dx = ?$

Sol

put $3x+2 = t \Rightarrow x = \frac{t-2}{3}$
 $3dx = dt$
 $dx = \frac{dt}{3}$

$$\Rightarrow \int \sin t \frac{dt}{3}$$

$$\Rightarrow \frac{1}{3} \int \sin t dt$$

$$\Rightarrow \frac{1}{3} (-\cos t) + C$$

$$\Rightarrow -\frac{\cos(3x+2)}{3} + C$$

Q 11 $\int \cos 2y dy$

Sol

$$\Rightarrow \int \cos 2y dy$$

put $2y = t$
 $2dy = dt$

$$dy = \frac{dt}{2}$$

$$\frac{dt}{2}$$

$$dt$$

$$t + C$$

$$\Rightarrow \frac{1}{2} \sin t + C$$

$$\Rightarrow \frac{1}{2} \sin 2y + C.$$

Q 3 (xii)

Q $\int \sec^2 2ax \, dx \Rightarrow$
 Sol

$$\Rightarrow \text{put } 2ax = t \Rightarrow \frac{t}{2a} = x$$

$$2a \, dx = dt$$

$$dx = \frac{dt}{2a}$$

$$\Rightarrow \int \frac{\sec^2 t \, dt}{2a}$$

$$\Rightarrow \frac{1}{2a} \int \sec^2 t \, dt$$

$$\Rightarrow \frac{1}{2a} \tan t + C.$$

$$\Rightarrow \frac{1}{2a} \tan 2ax + C.$$

Q 3.

Q (viii) $\int \frac{\sin x \, dx}{\cos x}$

Sol

$$\int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} \right) dx$$

$$\Rightarrow \int \frac{\sin x}{\cos x} \, dx + \int dx$$

$$\Rightarrow \int \tan x \, dx + x + C.$$

$$\Rightarrow \int \ln(\sec x) + x + C$$

$$Q.3 \int (1 + \tan x)^2 dx: \quad \because \{1 + \tan^2 x = \sec^2 x\}$$

$$\Rightarrow \int (1 + \tan^2 x + 2 \tan x) dx$$

$$\Rightarrow \int (\sec^2 x + 2 \tan x) dx$$

$$\Rightarrow \int \sec^2 x dx + 2 \int \tan x dx$$

$$\Rightarrow \sec x \tan x + 2 \ln |\sec x| + C$$

$$Q.3 (xiv) \int \frac{\sec x \cdot \tan x}{a + b \sec x} dx$$

$$\Rightarrow \int \frac{\sec x \tan x}{a + b \sec x} dx$$

$$\Rightarrow \frac{1}{b} \int \frac{b \sec x \tan x}{a + b \sec x} dx$$

$$\Rightarrow \frac{1}{b} \ln(a + b \sec x) + C$$

$$\Rightarrow \ln(a + b \sec x)^b + C$$

$$Q.3 (xvi) \int \frac{\sin \ln x}{x(3 - \cos \ln x)^{1/2}} dx$$

$$\Rightarrow \int (3 - \cos \ln x)^{-1/2} \left(\frac{\sin \ln x}{x} \right) dx$$

$$\Rightarrow \int 2(3 - \cos \ln x)^{1/2} + C$$

$$3 - \cos \ln x = y$$

$$0 - (-\sin \ln x) \frac{dx}{dx} = \frac{dy}{dx}$$

$$\frac{\sin \ln x}{x} = \frac{dy}{dx}$$

8.3

Q 3 (iii) $\int \sin^2 x \cos x \, dx$

Sol

Let $\sin x = u$

$$\cos x \, dx = du$$

$$dx = \frac{du}{\cos x}$$

$$\Rightarrow \int \frac{u^2 \cdot \cos x \, du}{\cos x}$$

$$\Rightarrow \int u^2 \, du$$

$$\Rightarrow \frac{u^3}{3} + C$$

$$\Rightarrow \frac{(\sin x)^3}{3} + C$$

Q 3 (v) $\int x \cdot \cos t x \, dx$

Sol

Put $x^2 = u$

$$2x \, dx = du$$

$$x \, dx = \frac{du}{2}$$

$$\frac{du}{2}$$

$$\Rightarrow \int \cos u \, \frac{du}{2}$$

$$\Rightarrow \frac{1}{2} \int \cos u \, du$$

$$= \frac{1}{2} \sin x + C.$$

$$\Rightarrow \frac{1}{2} \sin x + C$$

$$Q3(x) \int e^x \cos e^x dx$$

$$\text{Put } e^x = u$$

$$\frac{dx e^x}{du} du$$

$$\Rightarrow \int \cos u du$$

$$\Rightarrow \sin u + C$$

$$\Rightarrow \sin e^x + C$$

$$(1) \int x \cos x^2 dx$$

$$\int e^x \tan e^x dx$$

$$\int \cot(5x-9) dx$$

$$\int x^2 \sin x^3 dx$$

$$\int x^4 \tan x^5 dx$$

$$\int (1+x^2)^{2/5} x dx$$

$$\int \tan(3x-3) dx$$

$$\int \sec^2(4x+13) dx$$

$$\int e^x \sec^2 e^x dx$$

$$(10) \int \tan x \sec x dx$$

8.4

Q 1

$$\int_0^3 \sqrt[3]{(3t-1)^2} dt$$

$$\Rightarrow \int_0^3 \sqrt[3]{(3t-1)^2} dt$$

$$\sqrt[3]{8} = 2$$

$$\Rightarrow \frac{1}{3} \int_0^3 (3t-1)^{2/3} dt$$

$$\Rightarrow \frac{1}{3} \left[\frac{(3t-1)^{5/3}}{5/3} \right]_0^3$$

$$\Rightarrow \frac{1}{5} \left[(3t-1)^{5/3} \right]_0^3$$

$$8^{5/3} = 2^5$$

$$\Rightarrow \frac{1}{5} \left[(3(3)-1)^{5/3} - (3(0)-1)^{5/3} \right]$$

$$\Rightarrow \frac{1}{5} \left\{ 18^{5/3} - (-1)^{5/3} \right\}$$

$$\Rightarrow \frac{1}{5} \left\{ (8)^{5/3} - (-1)^{5/3} \right\}$$

$$\Rightarrow \frac{1}{5} \left\{ 2^5 + 1 \right\}$$

$$\Rightarrow \frac{1}{5} (32 + 1) \Rightarrow \frac{33}{5}$$

Q 1 $\int_{-2}^1 \sqrt{2-x} dx$

$$\Rightarrow \int_{-2}^1 \sqrt{2-x} dx$$

$$\Rightarrow -1 \int_{-2}^1 \sqrt{2-x} \left(\frac{-1}{-1} \right) dx$$

8.4

$$\Rightarrow -1 \int_{-2}^1 \sqrt{2-x} (-1) dx.$$

$$\Rightarrow -1 \left[\frac{(2-x)^{3/2}}{3/2} \right]_{-2}^1$$

$$\Rightarrow -2/3 \left[(2-x)^{3/2} \right]_{-2}^1$$

$$\Rightarrow -\frac{2}{3} \{ (2-1)^{3/2} - (2+2)^{3/2} \}.$$

$$\Rightarrow -\frac{2}{3} \{ 1^{3/2} - 4^{3/2} \}.$$

$$\Rightarrow -\frac{2}{3} \{ 1 - 8 \}$$

$$\Rightarrow -\frac{2}{3} (-7) = \frac{14}{3} \quad \text{Ans}$$

Q3 $\int \frac{dx}{(x-1)^3}$

Ans

done.

Q4 $\int_{-2}^1 x \sqrt{2x^2+3} dx$

sl

$$\int_{-2}^1 \sqrt{2x^2+3} \cdot x dx$$

$$\Rightarrow \frac{1}{4} \int_{-2}^1 (2x^2+3)^{1/2} \cdot 4x dx.$$

$$\Rightarrow \frac{1}{4} \left[\frac{(2x^2+3)^{1/2+1}}{3/2} \right]_{-2}^1$$

$$\Rightarrow \frac{1}{4} \times \frac{2}{3} \left\{ \{2(1)^2+3\}^{3/2} - \{2(-2)^2+3\}^{3/2} \right\}$$

$$\Rightarrow \frac{1}{6} \{ 5^{3/2} - 11^{3/2} \}$$

$$\Rightarrow \frac{1}{6} [5\sqrt{5} - 11\sqrt{11}]$$

Ans

Q5 $\int_0^5 \frac{2x+3}{\sqrt{x^2+3x+1}} dx$

Sol

$$\int_0^5 \frac{2x+3}{\sqrt{x^2+3x+1}} dx$$

$$\Rightarrow \int_0^5 (x^2+3x+1)^{-1/2} (2x+3) dx$$

$$\Rightarrow \left[\frac{(x^2+3x+1)^{1/2}}{1/2} \right]_0^5$$

$$\Rightarrow 2 \left[(x^2+3x+1)^{1/2} \right]_0^5$$

$$\Rightarrow 2 \left[\{ (5)^2+3(5)+1 \}^{1/2} - \{ 0+0+1 \}^{1/2} \right]$$

$$\Rightarrow 2 [\sqrt{41} - 1]$$

Q6 $\int_1^2 (2x+1)^3 \sqrt{x^2+x+1} dx$

Sol

$$\Rightarrow \int_1^2 (2x+1)^3 \sqrt{x^2+x+1} dx$$

8.11

①

Q Solve the differential equations.

Q. A equation contain the derivative of the function $f(x)$ is called differential equation.

Ex. ① $\frac{dy}{dx} = \cos x$

② $\frac{dy}{dx} = 2x^2 + 5$

③ $\frac{dy}{dx} = \cos x$

Variable separate form.

Q 3 $\frac{dy}{dx} = x + \sin x$

Sol. $y = 3$ when $x = 0$ condition
 $dy = (x + \sin x) dx$

Now we integrating both sides

$$\int dy = \int (x + \sin x) dx$$

$$y = \int x dx + \int \sin x dx$$

$$y = \frac{x^2}{2} - \cos x + C$$

Put $y = 3 ; x = 0$.

$$3 = 0 - \cos 0 + C$$

$$3 + 1 = C$$

$$C = 4$$

Now, Solution become

$$y = \frac{x^2}{2} - \cos x + 4$$

Q 8 $\frac{ds}{dt} = s^2 t \sqrt{1+t^2}$

Note:-
No condition

$$\frac{ds}{s^2} = dt + \sqrt{1+t^2}$$

$$s^{-2} ds = t \sqrt{1+t^2} dt$$

On integrating both sides

$$\int s^{-2} ds = \int t \sqrt{1+t^2} dt$$

$$\frac{s^{-1}}{-1} = \frac{1}{2} \cdot \frac{2}{3} (1+t^2)^{3/2} dt$$

$$-\frac{1}{s} = \frac{1}{3} (1+t^2)^{3/2} + C$$

Q 9 $\frac{du}{dx} = x^3 \sqrt{u}$

$$\frac{du}{\sqrt{u}} = x^3 dx$$

On integrating both sides

$$\int u^{-1/2} du = \int x^3 dx$$

$$2u^{1/2} = \frac{x^4}{4} + C$$

Q 10 $\frac{dv}{dt} = \sqrt{5v-7} (t \cos t^2)$

$$\frac{dx}{\sqrt{5x-7}} = (\sec^2 t) dt$$

On integrating both sides.

$$\int (5x-7)^{-1/2} dx = \int (\sec^2 t) dt$$

$$\frac{(5x-7)^{1/2}}{\frac{5}{2}} = \frac{1}{2} \int \sec^2 t \cdot 2t dt$$

$$\frac{2}{5} (5x-7)^{1/2} = \frac{1}{2} \tan t^2 + C$$

Q 11 $\int \frac{dy}{\sin^2 y} = \sec^2 x \sin x \cos x$

Ans $\frac{dy}{\sin^2 y} = \cos^2 x \sin x dx$

On integrating both sides.

$$\int \frac{dy}{\sin^2 y} = \int \cos^2 x \sin x dx$$

$$\int \frac{1}{\sin^2 y} dy = - \int \cos^2 x (-\sin x) dx$$

$$\int \csc^2 y dy = - \int \cos^2 x (-\sin x) dx$$

$$-\cot y = - \frac{\cos^3 x}{3} + C$$

$$\cot y = \frac{\cos^3 x}{3} + C$$

$$Q 13 \frac{dy}{dx} = \frac{\sin^2 y}{\cos^2 x}$$

$$\text{Sol} \frac{dy}{\sin^2 y} = \frac{\cos^2 x \cdot dx}{\cos^2 x}$$

$$\operatorname{cosec}^2 y \, dy = \sec^2 x \, dx$$

On integrating both sides.

$$\int \operatorname{cosec}^2 y \, dy = \int \sec^2 x \, dx$$

$$-\cot y = \tan x + c$$

$$0 = \tan x + \cot y + c$$

$$Q 14 \quad x^2 \frac{dy}{dx} = x^4 y^2 + y^2$$

$$\text{Sol} \quad \frac{dy}{y^2} = \frac{(x^4 + 1) \, dx}{x^2}$$

$$y^{-2} \, dy = (x^2 + x^{-2}) \, dx$$

On integrating both sides.

$$\int y^{-2} \, dy = \int (x^2 + x^{-2}) \, dx$$

$$\frac{y^{-1}}{(-1)} = \frac{x^3}{3} - \frac{x^{-1}}{1} + c$$

$$\frac{1}{y} = \frac{x^3}{3} - \frac{1}{x} + c$$

$$c = \frac{1}{y} + \frac{x^3}{3} - \frac{1}{x} + c$$

$$Q 15 \quad y(1+x^2) \frac{dy}{dx} = x(1+y^2)^2$$

$$\text{Sol} \quad \frac{y \, dy}{(1+y^2)^2} = \frac{x \, dx}{(1+x^2)}$$

On integrating

$$\int \frac{y}{(1+y^2)^2} dy = \int \frac{x^{-1}}{1+x^2} \quad (2)$$

$$\int \frac{y}{(1+y^2)^2} dy = \frac{1}{2} \ln(1+x^2) + C$$

$$\frac{1}{2} \int (1+y^2)^{-2} (2y) dy = \frac{1}{2} \ln(1+x^2) + C$$

$$\frac{1}{2} (1+y^2)^{-1} = \frac{1}{2} \ln(1+x^2) + C$$

$$-\frac{1}{2} (1+y^2) = \ln \sqrt{1+x^2} + C$$

Q. Ex $\frac{dy}{dx} = x^2 y^3$; $y=-1$; $x=1$

Sol.

$$\frac{dy}{dx} = x^2 y^3$$

$$\frac{dy}{y^3} = \int x^2 dx$$

On integ

$$\int y^{-3} dy = \int x^2 dx$$

$$\frac{y^{-2}}{-2} = \frac{x^3}{3} + C$$

$$-\frac{1}{2y^2} = \frac{x^3}{3} + C$$

Put $x=1$; $y=-1$

$$\int \frac{dy}{dx} = \frac{1}{2(-1)^2} = \frac{1}{2} + C$$

$$-\frac{1}{2} = -\frac{1}{3} + C$$

$$-\frac{1}{2} - \frac{1}{3} = C$$

$$-\frac{5}{6} = c$$

∴ Req. solution will be

$$-\frac{1}{2y} = \frac{1x^2 - 5}{3 \times 6}$$

$$Q 19 \quad \frac{ds}{dt} = \sqrt{s+1} \sqrt{3t+1}$$

sol

$$s=3; t=5$$

$$\frac{ds}{\sqrt{s+1}} = \sqrt{3t+1} dt$$

On integ b/s.

$$\int \frac{ds}{\sqrt{s+1}} = \int \sqrt{3t+1} dt$$

$$\frac{\sqrt{s+1}}{1/2} = \frac{1}{3} \frac{(3t+1)^{3/2}}{3/2} + c$$

$$2\sqrt{s+1} = \frac{2}{9} (3t+1)^{3/2} + c$$

Put $s=3; t=5$

$$2\sqrt{3+1} = \frac{2}{9} (3 \cdot 5 + 1)^{3/2} + c$$

$$2 \times 2 = \frac{2}{9} (16)^{3/2} + c$$

$$4 = \frac{2}{9} (64) + c$$

$$4 - \frac{2}{9} (64) = c$$

$$c = 36 - 128$$

$$c = -92$$

∴ req. solution will be

$$2\sqrt{s+1} = \frac{7}{9} (3t+1)^{3/2} - \frac{92}{9}$$

$$Q20 \frac{dx}{ds} = \frac{\sqrt{x^2+1} \sqrt{2s+3}}{x} \quad x(11) = \sqrt{3}$$

$$\int \frac{x dx}{\sqrt{x^2+1}} = \int \frac{ds \sqrt{2s+3}}{\sqrt{x^2+1}}$$

On integrating both sides.

$$\int \frac{x}{\sqrt{x^2+1}} dx = \int \sqrt{2s+3} ds$$

$$\frac{1}{2} \int (x^2+1)^{-1/2} 2x dx = \frac{1}{2} \cdot \frac{2}{3} (2s+3)^{3/2} + C$$

$$\frac{1}{2} (x^2+1)^{1/2} = \frac{1}{3} (2s+3)^{3/2} + C$$

$$(x^2+1)^{1/2} = \frac{2}{3} (2s+3)^{3/2} + C$$

Put $x = \sqrt{3}$
 $s = 11$

$$(3+1)^{1/2} = \frac{2}{3} (22+3)^{3/2} + C$$

$$2 = \frac{2}{3} \cdot 25\sqrt{25} + C$$

$$2 - \frac{1}{3}(125) = C$$

$$C = -125$$

$$C = -125$$

$$C = -\frac{119}{3}$$

∴ Req. solution is

$$(x^2+1)^{1/2} = \frac{2}{3} (2s+3)^{3/2} - \frac{119}{3}$$

$$Q(21) \frac{dy}{dx} = \frac{\sqrt{1+\cos y}}{\sin y}$$

Sl.

$$y(3) = \frac{\pi}{2}$$

$$\frac{dy}{\sin y} = \frac{\sqrt{1+\cos y}}{\sin y} dx$$

On Integrating

$$\Rightarrow \int \frac{\sin y}{\sqrt{1+\cos y}} dy = \int dx$$

$$\Rightarrow \int \frac{\sin y}{\sqrt{1+\cos y}} dy = \int dx$$

$$\Rightarrow \int (1+\cos y)^{-1/2} (-\sin y) dy = \int dx$$

$$\Rightarrow -\frac{(1+\cos y)^{1/2}}{1/2} = x + c$$

$$\Rightarrow -2(1+\cos y)^{1/2} = x + c$$

$$\text{Put } y = \frac{\pi}{2}$$

$$x = 3$$

$$\therefore -2(1+\cos \pi/2)^{1/2} = 3 + c$$

$$-2(1+0)^{1/2} = 3 + c$$

$$-2 - 3 = c$$

$$\Rightarrow c = -5$$

\therefore Req. solution will be

$$-2(1+\cos y)^{1/2} = x - 5$$

$$\sqrt{1+\cos y} = \frac{-x+5}{2}$$

Q 22 $y \frac{dy}{dx} = x(y^4 + 2y^2 + 1)$; $y(-3) = 1$ (3)

Sol

$$y \frac{dy}{dx} = x(y^4 + 2y^2 + 1)$$

$$\frac{y dy}{(y^4 + 2y^2 + 1)} = x dx$$

On integrating both sides

$$\int \frac{y dy}{(y^2 + 1)^2} = \int x dx$$

$$\frac{1}{2} \int (y^2 + 1)^{-2} (2y) dy = \frac{x^2}{2} + C$$

$$\frac{(y^2 + 1)^{-1}}{-1} = \frac{x^2}{2} + C$$

$$-\frac{1}{2(1+y^2)} = \frac{x^2}{2} + C$$

Put $y = 1$, $x = -3$

$$-\frac{1}{2(1+1)} = \frac{9}{2} + C$$

$$-\frac{1}{4} = \frac{9}{2} + C$$

$$-\frac{1}{4} - \frac{9}{2} = C$$

$$-\frac{1-18}{4} = C$$

$$C = -\frac{17}{4}$$

\therefore Req. solution will be

$$-\frac{1}{2(y^2+1)} = -\frac{x}{2} + \frac{19}{4}$$

Q 24 ^{Imp} $\frac{dw}{dz} = \sqrt{wz-2w-3z+6}$

$w=12$
 $z=6$

$$\frac{dw}{dz} = \sqrt{w(z-2)-3(z-2)}$$

$$\frac{dw}{dz} = \sqrt{(w-3)(z-2)}$$

$$\frac{dw}{\sqrt{w-3}} = (z-2)^{1/2} dz$$

On integrating both sides

$$\int \frac{dw}{\sqrt{w-3}} = \int (z-2)^{1/2} dz$$

$$\int (w-3)^{-1/2} dw = \int (z-2)^{1/2} dz$$

$$2(w-3)^{1/2} = \frac{2}{3}(z-2)^{3/2} + C$$

Put $w=12$
 $z=6$

$$\therefore 2(12-3)^{1/2} = \frac{2}{3}(6-2)^{3/2} + C$$

$$2 \times 3 = \frac{2}{3} 4\sqrt{4} + C$$

$$6 - \frac{16}{3} = C$$

$$C = \frac{2}{3}$$

\therefore Req solution will be

$$2\sqrt{w-3} = \frac{2}{3} (z-2)^{3/2+2/3}$$

$$\sqrt{w-3} = \frac{(z-2)^{3/2}}{3} + \frac{1}{3}$$

$$\sqrt{w-3} = \frac{1}{3} \{ (z-2)^{3/2} + 1 \}$$

Q25 $2 + 2y \frac{dy}{dx} = 1 + 3x^2$ $y(2) = 1$

$$2(1+2y) \frac{dy}{dx} = 1 + 3x^2 + 2$$

$$2y \frac{dy}{dx} = 3x^2 - 1$$

$$2y dy = (3x^2 - 1) dx$$

On integrating both sides.

$$\int 2y dy = \int (3x^2 - 1) dx$$

$$\frac{2y^2}{2} = \frac{3x^3}{3} - x + C$$

$$y^2 = x^3 - x + C$$

Put $x = 2$; $y = 1$

$$\therefore 1^2 = 2^3 - 2 + C$$

$$1 + 2 - 8 = C$$

$$C = -5$$

\therefore The req solution will be

$$y^2 + x^3 - x - 5$$

Q.5

$$Q 9 \int_0^1 \frac{dx}{\sqrt{4-x^2}}$$

$$\Rightarrow \int_0^1 \frac{dx}{\sqrt{(2)^2 - x^2}}$$

Put $x = 2 \sin \theta$ when $x = 0$
 $dx = 2 \cos \theta d\theta$ $0 = 2 \sin \theta$
 $\theta = 0$

$$x = 1$$

$$1 = 2 \sin \theta$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \pi/6$$

$$\Rightarrow \int_0^{\pi/6} \frac{2 \cos \theta d\theta}{\sqrt{4 - 4 \sin^2 \theta}}$$

$$\Rightarrow \int_0^{\pi/6} \frac{2 \cos \theta d\theta}{2 \cos \theta}$$

$$\Rightarrow \int_0^{\pi/6} d\theta$$

$$\Rightarrow \left[\theta \right]_0^{\pi/6}$$

$$\Rightarrow \left[\pi/6 - 0 \right]$$

$$\Rightarrow \left[\pi/6 \right]$$

$$Q9 \int_{\sqrt{3}}^{2\sqrt{3}} \frac{dx}{x^2 \sqrt{x^2 + 9}}$$

$$\Rightarrow \int_{\sqrt{3}}^{3\sqrt{3}} \frac{dx}{x^2 \sqrt{x^2 + (3)^2}}$$

$$\Rightarrow \text{Put } x = 3 \tan \theta$$

$$dx = 3 \sec^2 \theta d\theta$$

When

$$x = \sqrt{3}$$

$$\sqrt{3} = \sqrt{3} \cdot \sqrt{3} \tan \theta$$

$$\theta = \tan^{-1} \frac{1}{\sqrt{3}}$$

$$\theta = \pi/6$$

$$3\sqrt{3} = 3 \tan \theta$$

$$\theta = \pi/3$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{3 \sec^2 \theta d\theta}{(3 \tan^2 \theta) \sqrt{9 \tan^2 \theta + 9}}$$

$$\frac{3 \sec^2 \theta}{1} \rightarrow 10$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{3 \sec^2 \theta d\theta}{9 \tan^2 \theta \cdot 3 \sec \theta}$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{\sec \theta d\theta}{9 \tan^2 \theta}$$

$$\Rightarrow \frac{1}{9} \int_{\pi/6}^{\pi/3} \frac{\sec \theta d\theta}{\tan^2 \theta}$$

$$\Rightarrow \frac{1}{9} \int_{\pi/6}^{\pi/3} \frac{\sec \theta \cos^2 \theta d\theta}{\sin^2 \theta}$$

$$\rightarrow \frac{1}{9} \int_{\pi/6}^{\pi/3} \frac{1}{\cos \theta} \frac{\cos 2\theta}{\sin 2\theta} d\theta.$$

$$\Rightarrow \frac{1}{9} \int_{\pi/6}^{\pi/3} \sin^2 \theta \cos \theta d\theta.$$

$$\Rightarrow \frac{1}{9} \left[\frac{\sin^{-1} \theta}{-1} \right]_{\pi/6}^{\pi/3}$$

$$\Rightarrow -\frac{1}{9} \left[\frac{1}{\sin \theta} \right]_{\pi/6}^{\pi/3}$$

$$\Rightarrow -\frac{1}{9} \left[\frac{1}{\sin \pi/3} - \frac{1}{\sin \pi/6} \right]$$

$$\Rightarrow -\frac{1}{9} \left[\frac{2}{\sqrt{3}} - \frac{1}{\frac{1}{2}} \right]$$

$$\Rightarrow -\frac{1}{9} \left[\frac{2}{\sqrt{3}} - 2 \right]$$

$$\Rightarrow -\frac{1}{9} \left[\frac{2 - 2\sqrt{3}}{\sqrt{3}} \right]$$

$$\Rightarrow -\frac{1}{9\sqrt{3}} [2 - 2\sqrt{3}]$$

$$\text{Q 11} \int \frac{x^3 dx}{\sqrt{a^2 - x^2}}$$

$$\text{Put } x = a \sin \theta$$

$$dx = a \cos \theta d\theta.$$

$$\Rightarrow \int \frac{(a \sin \theta)^3 a \cos \theta d\theta}{\sqrt{a^2 - a^2 \sin^2 \theta}}$$

$$\Rightarrow \int \frac{a^3 \sin^3 \theta a \cos \theta d\theta}{a \cos \theta}$$

$$\Rightarrow a^3 \int \sin^3 \theta d\theta$$

$$\Rightarrow a^3 \int \sin^2 \theta \sin \theta d\theta$$

$$\Rightarrow a^3 \int (1 - \cos^2 \theta) \sin \theta d\theta$$

$$\Rightarrow a^3 \int \sin \theta d\theta - \int \cos^2 \theta \sin \theta d\theta$$

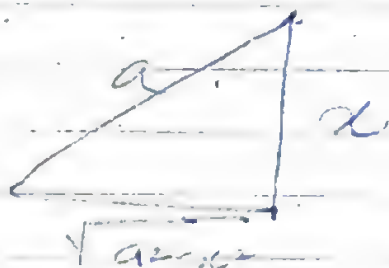
$$\Rightarrow a^3 \int \sin \theta d\theta + a^3 \int \cos^2 \theta (-\sin \theta) d\theta$$

$$\Rightarrow -a^3 \cos \theta - \frac{a^3 \cos^3 \theta}{3} + C$$

$$\Rightarrow \text{Ans}$$

$$x = a \sin \theta$$

$$\frac{x}{a} = \sin \theta$$



$$\Rightarrow -a^2 \sqrt{a^2 - x^2} + \frac{a^3}{3} \left(\frac{\sqrt{a^2 - x^2}}{a} \right)^3 + C$$

$$\Rightarrow -a^2 \sqrt{a^2 - x^2} + \frac{a^3}{3} \left(\frac{\sqrt{a^2 - x^2}}{a} \right)^3 + C$$

$$\Rightarrow -a^2 \sqrt{a^2 - x^2} + \frac{1}{3} (a^2 - x^2)^{3/2} + C$$

Ans

Trigonometric substitution

① $\sqrt{a^2 - x^2}$

$$\text{Q } \int_{-1}^{\sqrt{3}} \sqrt{4 - x^2} \, dx$$
$$\Rightarrow \int_{-1}^{\sqrt{3}} \sqrt{(2)^2 - x^2} \, dx$$

$$\text{Put } x = 2 \sin \theta$$

$$dx = 2 \cos \theta \, d\theta$$

When

$$x = -1 \Rightarrow -1 = 2 \sin \theta$$

$$\theta = \sin^{-1}\left(\frac{-1}{2}\right)$$

$$\theta = -\pi/6$$

$$x = \sqrt{3} \Rightarrow \sqrt{3} = 2 \sin \theta$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = \pi/3$$

$$\therefore \int_{-\pi/6}^{\pi/3} \sqrt{4 - 4 \sin^2 \theta} \, 2 \cos \theta \, d\theta$$

$$\Rightarrow 2 \int_{-\pi/6}^{\pi/3} \sqrt{1 - \sin^2 \theta} \, 2 \cos \theta \, d\theta$$

$$\Rightarrow 4 \int_{-\pi/6}^{\pi/3} \cos^2 \theta \, d\theta$$

$$\Rightarrow \int_{-\pi/6}^{\pi/3} (1 + \frac{\cos 2\theta}{2}) \, d\theta \quad \because \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\Rightarrow \frac{4}{2} \int_{-\pi/6}^{\pi/3} (1 + \cos 2\theta) \, d\theta$$

$$\Rightarrow 2 \left[\theta + \frac{\sin \theta}{2} \right]_{-\pi/6}^{\pi/3}$$

$$\Rightarrow 2 \left\{ \left[\frac{\pi}{3} + \frac{\sin^2 \pi/3}{2} \right] - \left[-\frac{\pi}{6} + \frac{\sin^2(\pi/6)}{2} \right] \right\}$$

$$\Rightarrow 2 \left\{ \left[\frac{\pi}{3} + \frac{\sqrt{3}}{4} \right] - \left[-\frac{\pi}{6} + \frac{(-\sqrt{3}/2)}{2} \right] \right\}$$

$$\Rightarrow 2 \left\{ \left[\frac{\pi}{3} + \frac{\sqrt{3}}{4} + \frac{\pi}{6} + \frac{\sqrt{3}}{4} \right] \right\}$$

$$\Rightarrow 2 \left\{ \frac{4\pi + 3\sqrt{3} + 2\pi + 3\sqrt{3}}{2} \right\}$$

$$\Rightarrow \frac{6\pi + 6\sqrt{3}}{2} = \frac{6(\pi + \sqrt{3})}{2}$$

$$\Rightarrow \pi + \sqrt{3}$$

$$\int_0^{\pi/4} \cos^4 2x \sin^3 2x dx$$

$$\Rightarrow \int \cos^4 2x \cdot \sin^3 2x dx$$

$$\Rightarrow \int \cos^4 2x \sin^2 2x \sin 2x dx$$

$$\Rightarrow \int \cos^4 2x (1 - \cos^2 2x) \sin 2x dx$$

$$\Rightarrow \int \cos^4 2x (1 - \cos^2 2x) \sin 2x dx$$

same
Q $\int \sec^4 2u \, du = ?$

Q $\int \tan^4 x \, dx$
sol

$$\Rightarrow \int \tan^2 x \tan^2 x \, dx$$

$$\Rightarrow \int \tan^2 x (\sec^2 x - 1) \, dx$$

$$\Rightarrow \int \tan^2 x \sec^2 x - \int \tan^2 x \, dx$$

$$\Rightarrow \int \tan^2 x \sec^2 x \, dx - \int (\sec^2 x - 1) \, dx$$

$$\Rightarrow \int \tan^2 x \sec^2 x \, dx - \int \sec^2 x \, dx + \int dx$$

$$\Rightarrow \int \frac{\tan^{2+1} x}{3} - \tan x + x + C$$

$$\Rightarrow \frac{1}{3} \tan^3 x - \tan x + x + C$$

Q $\int \sec^4 2u \, du = ?$

sol $\int (\sec^2 2u)^2 \, du$

$$\Rightarrow \int \sec^2 2u \sec^2 2u \, du$$

$$\Rightarrow \int \sec^2 2u (1 + \tan^2 2u) \, du$$

$$\Rightarrow \int \sec^2 2u \, dx + \int \sec^2 2u \tan^2 2u \, du$$

$$\Rightarrow \frac{\tan 2u}{2} + \frac{\tan^3 2u}{3 \times 2} + C$$

$$\frac{1}{2} du = ?$$

$$\frac{1}{2} \tan u + \frac{1}{6} \frac{\tan^3 u}{3} + C$$

$$Q \int \tan^3 x \, dx$$

$$\Rightarrow \int \tan^2 x \tan x \, dx$$

$$\Rightarrow \int \tan^2 x (\sec^2 x - 1) \, dx$$

$$\Rightarrow \int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx$$

$$\Rightarrow \int \tan^2 x \sec^2 x \, dx - \int \tan x \tan x \, dx$$

$$\Rightarrow \int \tan^2 x \sec^2 x \, dx - \int \tan x (\sec x - 1) \, dx$$

$$\Rightarrow \int \tan^2 x \sec^2 x \, dx - \int \tan x \sec x \, dx + \int \tan x \, dx$$

$$\Rightarrow \frac{\tan^3 x}{3} - \frac{\tan^2 x}{2} + \ln |\sec x| + C$$

$$Q \int \cos 2x \, dx$$

$$\Rightarrow \frac{\sin 2x}{2} + C$$

$$Q \int \sec^2 x \, dx$$

$$\tan x \, dx$$

$$C$$



$$Q18 \int \tan^3 3x \cdot \sec^4 3x dx$$

~~sol~~

$$\text{put } 3x = u.$$

$$\Rightarrow 3dx = du.$$

$$\Rightarrow \frac{1}{3} \int \tan^3 u \sec^4 u du.$$

$$\Rightarrow \frac{1}{3} \int \tan^3 u \sec^2 u \sec^2 u du.$$

$$\Rightarrow \frac{1}{3} \int \tan^3 u (1 + \tan^2 u) \sec^2 u du$$

$$\Rightarrow \frac{1}{3} \int \tan^3 u \sec^2 u du + \frac{1}{3} \int \tan^5 u \sec^2 u du$$

$$\Rightarrow \frac{1}{3} \frac{\tan^4 u}{4} + \frac{1}{3} \frac{\tan^6 u}{6} + c$$

$$\Rightarrow \frac{1}{12} \tan^4 u + \frac{1}{18} \tan^6 u + c$$

$$Q \int \tan^3 2x \cdot \sec^3 2x dx$$

~~sol~~

$$\int \tan^3 2x \cdot \sec^3 2x dx$$

$$\text{Put } 2x = u$$

$$2 dx = du$$

$$dx = \frac{du}{2}$$

$$\Rightarrow \frac{1}{2} \int \tan^3 u \sec^3 u du$$

$$\Rightarrow \frac{1}{2} \int \tan^2 u \cdot \tan u \cdot \sec^3 u \cdot \sec u du$$

Q cot

$$\Rightarrow \frac{1}{2} \int \tan u (\sec^2 u - 1) \sec^3 u \, du$$

$$\Rightarrow \frac{1}{2} \int \tan u \sec^5 u \, du - \frac{1}{2} \int \tan u \sec^3 u \, du$$

$$\Rightarrow \frac{1}{2} \int \sec^4 u \tan u \sec^2 u \, du - \frac{1}{2} \int \sec^2 u \sec^2 u \tan u \, du$$

$$\Rightarrow \frac{1}{2} \cdot \frac{\sec^5 u}{5} - \frac{1}{2} \cdot \frac{\sec^3 u}{3} + C$$

$$\Rightarrow \frac{1}{10} \sec^5 2x - \frac{1}{6} \sec^3 2x + C$$

Q (xiii) $\int \cot^4 3z \, dz$

Put $3z = u$
 $3dz = du$

$$\Rightarrow \frac{1}{3} \int \cot^4 u \, du$$

$$\Rightarrow \frac{1}{3} \int \cot^2 u \cot^2 u \, du$$

$$\Rightarrow \frac{1}{3} \int \cot^2 u (\csc^2 u - 1) \, du$$

$$\Rightarrow \frac{1}{3} \int \cot^2 u \csc^2 u \, du - \frac{1}{3} \int \cot^2 u \, du$$

$$\Rightarrow \frac{1}{3} \int \cot^2 u \csc^2 u \, du - \frac{1}{3} \int (\csc^2 u - 1) \, du$$

$$\Rightarrow \frac{1}{3} \int \cot^2 u \csc^2 u \, du - \frac{1}{3} \csc^2 u + \frac{1}{3} \int du$$

$$\Rightarrow \frac{1}{3} \int \cot^2 u (-\cot u) \, du - \frac{1}{3} \csc^2 u + \frac{1}{3} \int du$$

$$\Rightarrow -\frac{1}{3} \frac{\cot^3 u}{3} - \frac{(-\cot^2 u)}{3} - \frac{1}{3} u$$

$\sec^3 u \, du$

$$= -\frac{1}{9} \cot^3 u + \frac{\cot^2 u}{2} + \frac{1}{3} u$$

$\tan u \sec^3 u \, du$

$$\Rightarrow \frac{1}{9} \cot^3 3\frac{\pi}{6} + \frac{1}{2} \cot^2 3\frac{\pi}{6} + \frac{1}{3} (3\frac{\pi}{6})$$

$\frac{1}{2} \int \sec u^2 \sec u \tan u \, du$

Ans 24.5

$3u + c$

8.1.

$$Q. 4 \int_{\pi/3}^{\pi/6} \sin^2 x \cos x \, dx$$

$$\Rightarrow \left[-\frac{\sin^3 x}{3} \right]_{\pi/3}^{\pi/6}$$

$$\Rightarrow \frac{1}{3} \left\{ (\sin \pi/6)^3 - (\sin \pi/3)^3 \right\}$$

$$\Rightarrow \frac{1}{3} \left\{ \left(\frac{1}{2}\right)^3 - \left(\frac{\sqrt{3}}{2}\right)^3 \right\}$$

$$\Rightarrow \frac{1}{3} \left\{ \frac{1}{8} - \frac{3\sqrt{3}}{8} \right\}$$

$$\Rightarrow \frac{1}{3} \left\{ \frac{1-3\sqrt{3}}{8} \right\}$$

$$= \frac{1}{24} (1-3\sqrt{3}) \text{ Ans}$$

$$Q. 9 \int_0^{\pi/2} \cos^4 x \, dx$$

$$\Rightarrow \int_0^{\pi/2} \cos^2 (1 - \sin^2 x) \, dx$$

$2x + c$

Ans

du

$(u-1) \, du$

$du - \frac{1}{3} \int \cot^2 u \, du$

$\frac{1}{3} \int (\sec u - 1) \, du$

$\sec u + \frac{1}{3} \int du$

$\sec^3 u + \frac{1}{3} \int du$

$$\Rightarrow \int_0^{\pi/2} \cos^2 x \, dx - \int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$$

Q10 $\int_{\pi/6}^{\pi/2} \frac{\cos 3x \, dx}{\sin x}$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos 3x \cdot \sin^{-1} x \, dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos^2 x \cos x \sin^{-1} x \, dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos^2 x (1 - \sin^2 x) \sin^{-1} x \, dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos x \sin^{-1} x \, dx - \int_{\pi/6}^{\pi/2} \sin^{3/2} x \, dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \cos x \sin^{-1} x \, dx - \int_{\pi/6}^{\pi/2} \sin^{3/2} x \, dx$$

$$\Rightarrow \left[\frac{\sin^{1/2} x}{1/2} \right]_{\pi/6}^{\pi/2} - \left[-\cos^{3/2} x \right]_{\pi/6}^{\pi/2}$$

$$\Rightarrow \frac{1}{2} \left\{ (\sin \pi/2)^{3/2} - (\sin \pi/6)^{3/2} \right\} + \left[(\cos \pi/2)^{3/2} - (\cos \pi/6)^{3/2} \right]$$

$$\Rightarrow \frac{1}{2} \left[0 - \left(\frac{\sqrt{3}}{2} \right)^{3/2} \right] + \left[0 - \left(\frac{1}{2} \right)^{3/2} \right]$$

$$\Rightarrow \frac{1}{2} \left\{ -\frac{3}{\sqrt{2}} \right\} + \left\{ -\frac{1}{4} \right\}$$

$$\Rightarrow -\frac{3}{2\sqrt{2}} - \frac{1}{4}$$

$$\Rightarrow \frac{-3(-4) - 1(2\sqrt{2})}{2\sqrt{2}(-4)} = \frac{12 - 2\sqrt{2}}{-8\sqrt{2}}$$

Q 10 $\int_{\pi/6}^{\pi/2} \frac{\cos^3 x}{\sqrt{\sin x}} dx = ?$

$$\int_{\pi/6}^{\pi/2} \cos^2 x \cos x (\sin x)^{-1/2} dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} (1 - \sin^2 x) \sin^{-1/2} x \cos x dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} (\sin^{-1/2} x - \sin^{3/2} x) \cos x dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/2} \sin^{-1/2} x \cos x dx - \int_{\pi/6}^{\pi/2} \sin^{3/2} x \cos x dx$$

$$\Rightarrow \left[\frac{\sin^{1/2} x}{1/2} \right]_{\pi/6}^{\pi/2} - \left[\frac{\sin^{5/2} x}{5/2} \right]_{\pi/6}^{\pi/2}$$

$$\Rightarrow \frac{2}{1} \left[\sin^{1/2} x \right]_{\pi/6}^{\pi/2} - \frac{2}{5} \left[\sin^{5/2} x \right]_{\pi/6}^{\pi/2}$$

$$\Rightarrow 2 \left[(\sin \pi/2)^{5/2} - (\sin \pi/6)^{5/2} \right] - \frac{2}{5} \left[(\sin \pi/2)^{7/2} - (\sin \pi/6)^{7/2} \right]$$

$$\Rightarrow 2 \left[(1)^{5/2} - \left(\frac{1}{2}\right)^{5/2} \right] - \frac{2}{5} \left[(1)^{7/2} - \left(\frac{1}{2}\right)^{7/2} \right]$$

$$\Rightarrow 2 - \frac{2}{\sqrt{2}} - \frac{2}{5} + \frac{2 \cdot 1}{5 \cdot 2\sqrt{2}}$$

$$\Rightarrow 2 - \sqrt{2} - \frac{2}{5} + \frac{\sqrt{2}}{5 \cdot 2\sqrt{2}}$$

$$\Rightarrow 2 - \sqrt{2} - \frac{2}{5} + \frac{1}{10\sqrt{2}}$$

$$\Rightarrow \left[\frac{8}{5} - \sqrt{2} - \frac{1}{10\sqrt{2}} \right] \underline{\underline{\Delta}}$$

Q11 $\int_0^{\pi/4} \tan^3 x \sec x dx$

$$\Rightarrow \int_0^{\pi/4} (\sec^2 x - 1) \tan x \sec x dx$$

$$\Rightarrow \int_0^{\pi/4} (\sec^3 x - \sec x) \tan x dx$$

$$\Rightarrow \int_0^{\pi/4} \sec^3 x \tan x dx - \int_0^{\pi/4} \sec x \tan x dx$$

$$\Rightarrow \int_0^{\pi/4} \sec^2 x \sec x \tan x dx - \int_0^{\pi/4} \sec x \tan x dx$$

$$\Rightarrow \left[\frac{\sec^3 x}{3} \right]_0^{\pi/4} - \left[\sec x \right]_0^{\pi/4}$$

$$\Rightarrow \frac{1}{3} \left[\sec^3 x \right]_0^{\pi/4} - \left[\sec x \right]_0^{\pi/4}$$

$$\cos 180^\circ = -1$$

$$\therefore \sec \pi = -1$$

$$\Rightarrow \frac{1}{3} \left[(\sec \pi)^3 - (\sec 0)^3 \right] - \left[(\sec \pi) - (\sec 0) \right]$$

$$\Rightarrow \frac{1}{3} \left[(-1)^3 - (1)^3 \right] - \left[-1 - 1 \right]$$

$$\Rightarrow + \frac{1(-2)}{3} + 2$$

$$\Rightarrow -\frac{2}{3} + 2$$

$$\Rightarrow \boxed{\frac{4}{3}} \text{ Ans}$$

$$Q \int_0^{\pi/4} \sin^2 2x \cos^2 2x dx = ?$$

$$\Rightarrow \int_0^{\pi/4} \sin^2 2x \cos^2 2x dx$$

Let

$$2x = u$$

$$2dx = du$$

$$\Rightarrow \frac{1}{2} \int_0^{\pi/4} \sin^2 u \cos^2 u du$$

$$\Rightarrow \frac{1}{2} \int_0^{\pi/4} \left(\frac{1 - \cos 2u}{2} \right) \left(\frac{1 + \cos 2u}{2} \right) du$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} (1 - \cos^2 2u) du$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} du - \frac{1}{8} \int_0^{\pi/4} \cos^2 2u du$$

$$\Rightarrow \frac{1}{8} \left[u \right]_0^{\pi/4} - \frac{1}{8} \left[\frac{u}{2} + \frac{\sin 2u}{4} \right]_0^{\pi/4}$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} du - \frac{1}{8} \int_0^{\pi/4} \left(\frac{1 + \cos 4u}{2} \right) du$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} du - \frac{1}{16} \int_0^{\pi/4} du -$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} \left(1 - \frac{1 - \cos 4u}{2}\right) du$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} \left(\frac{2 - 1 - \cos 4u}{2}\right) du$$

$$\Rightarrow \frac{1}{8} \int_0^{\pi/4} \left(\frac{1 - \cos 4u}{2}\right) du$$

$$\Rightarrow \frac{1}{16} \int_0^{\pi/4} du - \frac{1}{16} \int_0^{\pi/4} \cos 4u du$$

$$\Rightarrow \frac{1}{16} \left[u \right]_0^{\pi/4} - \frac{1}{16} \left[\frac{\sin 4u}{4} \right]_0^{\pi/4}$$

$$\Rightarrow \frac{1}{16} \left[u \right]_0^{\pi/4} - \frac{1}{64} \left[\sin 4u \right]_0^{\pi/4}$$

$$\Rightarrow \frac{1}{16} \left[\frac{\pi}{4} \right]$$

$$\Rightarrow \frac{1}{16} \left[2x \right]_0^{\pi/4} - \frac{1}{64} \left[\sin 8x \right]_0^{\pi/4}$$

$$\Rightarrow \frac{1}{16} \left[2\pi \right]$$

Q.7.

①

Integration by Parts:-

$$\int (I)(II) dx = (I) \cdot \left(\int \frac{II}{II} \right) - \left(\frac{Derivative}{bI} \right) \left(\frac{Next}{bII} \right) dx$$

$$\int u \cdot v dx = u \int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$$

Q. $\int x \sin x dx$

Using by parts.

$$\int u \cdot v dx = u \int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$$

$$\Rightarrow x(-\cos x) - \int \left\{ \frac{d(x)}{dx} \int \sin x dx \right\} dx$$

$$\Rightarrow -x \cos x - \int \{(-\cos x)\} dx$$

$$\Rightarrow -x \cos x + \int \cos x dx$$

$$\Rightarrow -x \cos x + \sin x + C \quad \text{Ans}$$

Q. $\int x \sin x dx$

Using parts:-

$$\int u \cdot v dx = u \int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$$

Put $u = x$ and $v = \sin x$.

$\frac{du}{dx} = 1$ and $\int v dx = \int \sin x dx = -\cos x$.

$$\int x \sin x dx = x \cdot (-\cos x) - \int 1 \cdot (-\cos x) dx$$

Liate.

L = \ln
I = Inverse Trig
A = Algebraic
T = Trigonometric
e = e^x

Note:- Inverse Trigonometric function and logarithmic function can be taking as second function in by parts formula.

$$\begin{aligned} Q \int \ln x \, dx \\ \Rightarrow \int 1 \cdot \ln x \, dx \\ \Rightarrow \int \ln x \cdot 1 \, dx \end{aligned}$$

$$\text{Using } \int u \cdot v \, dx = u \int v \, dx - \int \left\{ \frac{du}{dx} \int v \, dx \right\} dx$$

Taking

$$\begin{aligned} u &= \ln x & ; & \quad v = 1 \\ \frac{du}{dx} &= \frac{1}{x} & ; & \quad \int v \, dx = \int dx \\ & & \therefore & \quad \int v \, dx = x \end{aligned}$$

$$\begin{aligned} \therefore \int \ln x \cdot 1 \, dx &= \ln x \cdot x - \int \left(\frac{1}{x} \cdot x \right) dx \\ &= \ln x \cdot x - \int dx \end{aligned}$$

$$\int \ln x \, dx = x \ln x - x + C$$

Q.7

$$\int x \ln x \, dx$$

$$\rightarrow \int \ln x \cdot x \, dx$$

$$\Rightarrow \int \ln x \cdot x \, dx$$

Let

$$u = \ln x \quad v = x$$

$$\frac{du}{dx} = \frac{1}{x} ; \int v \, dx = \int x \, dx = \frac{x^2}{2}$$

Using by parts.

$$\int \ln x \cdot x \, dx = \ln x \left(\frac{x^2}{2} \right) - \int \left(\frac{1}{x} \cdot \frac{x^2}{2} \right) dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$$

$$\Rightarrow \frac{x^2}{2} \ln x - \frac{1}{2} \cdot \frac{x^2}{2} + C$$

$$\Rightarrow \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

Q.8

$$\int x^2 \tan^{-1} x \, dx$$

Using by parts: $\int u \cdot v \, dx = u \int v \, dx - \int \left\{ \frac{du}{dx} \int v \, dx \right\} dx$

$$u = \tan^{-1} x$$

$$\frac{du}{dx} = \frac{1}{1+x^2}$$

$$v = x^2$$

$$\int v \, dx = \int x^2 \, dx = \frac{1}{3} x^3$$

$$\int x^2 \tan^{-1} x \, dx = \tan^{-1} x \left(\frac{x^3}{3} \right) - \int \frac{1}{1+x^2} \left(\frac{x^3}{3} \right) dx$$

$$\Rightarrow \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \frac{x^3}{1+x^2} \, dx$$

Q4, 5, 6, 7, 8,

$$\frac{x^3}{1+x^2} \div \frac{x^3+1}{-1}$$

$$\Rightarrow \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \left(x - \frac{x}{1+x^2} \right) dx$$

$$\Rightarrow \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int x dx + \frac{1}{3} \int \frac{x}{1+x^2} dx$$

$$\Rightarrow \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int x dx + \frac{1}{3 \times 2} \int \frac{2x}{1+x^2} dx$$

$$\Rightarrow \frac{x^3}{3} \tan^{-1} x - \frac{1}{3 \times 2} x^2 + \frac{1}{6} \ln(1+x^2) + C$$

Q 1 (ii). $\int x \sin^2 x \cos x dx$

Ans

Let $u = x$

$\frac{du}{dx} = 1$

$v = \sin^2 x \cos x$

$\int v dx = \int \sin^2 x \cos x dx$

Using by parts: $\int v dx = \frac{\sin^3 x}{3}$

$$\int u \cdot v dx = u \int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$$

$$= \int x \sin^2 x \cos x dx = x \frac{\sin^3 x}{3} - \int \left(1 \cdot \frac{\sin^3 x}{3} \right) dx$$

$$= \frac{x}{3} \sin^3 x - \frac{1}{3} \int \sin^3 x dx$$

②

$$\int 1 \cdot dx \dots$$

$$Q \int (\ln x)^2 dx$$

$$\Rightarrow \int (\ln x)^2 \cdot 1 dx$$

Let

$$u = (\ln x)^2 \quad ; \quad v = 1$$

$$\frac{du}{dx} = 2 \ln x \cdot \frac{1}{x}$$

$$f dx = f \cdot dx = x$$

Using

$$\int u \cdot v dx = u \int v dx - \int \left\{ \frac{du}{dx} \cdot f dx \right\} dx$$

$$\int (\ln x)^2 \cdot 1 dx = (\ln x)^2 \cdot x - \int \left(\frac{2 \ln x \cdot x}{x} \right) dx$$

$$= (\ln x)^2 \cdot x - 2 \int \ln x dx$$

$$= (\ln x)^2 \cdot x - 2 \left[\frac{\ln x \cdot x}{x} - \int \left(\frac{1}{x} \right) x dx \right]$$

$$= (\ln x)^2 \cdot x - 2 [x \ln x - x] + c$$

$$= (\ln x)^2 \cdot x - 2x \ln x + 2x + c$$

Theorem = Every integral can be solved by integration by parts.

$$Q^2 \text{ (i)} \int \sqrt{4-x^2} dx$$

$$\Rightarrow \int 1 \sqrt{4-x^2} dx$$

\Rightarrow Let

$$u = 1$$

$$v = \sqrt{4-x^2}$$

$$\frac{du}{dx} = 0$$

$$f v dx = \int \sqrt{4-x^2} du$$

Using

$$\int u \cdot v dx = u \cdot v dx - \int \left\{ \frac{du}{dx} f dx \right\} \cdot dx$$

$$\Rightarrow 1 \cdot \int \sqrt{4-x^2} du$$

$$Q \int x^2 e^x dx = ?$$

Sol Let

$$u = x^2$$

$$; v = e^x$$

$$\frac{du}{dx} = 2x$$

$$\int v dx = \int e^x dx = e^x$$

$$Q \int x e^x dx$$

$$\text{Let } u = x \quad , \quad v = e^x$$

$$\frac{du}{dx} = 1 \quad \int v du = \int e^x dx = e^x$$

$$\int x e^x dx = x e^x - \int 1 \cdot e^x dx$$

$$= x e^x - \int e^x dx$$

$$= x e^x - e^x + C$$

$$= e^x (x - 1) + C$$

$$Q \int \sqrt{\sin 2x} \cdot 2x \cdot dx$$

$$\text{Put } \sqrt{2x} = t$$

$$2x = t^2$$

$$2t \frac{dt}{dx} = 2$$

$$t \cdot dt = dx$$

$$\Rightarrow \int \sin t \cdot t \cdot dt$$

$$\Rightarrow \int t \sin t \cdot dt$$

Let.

$$t = u$$

$$\frac{dt}{du} = 1$$

$$v = \sin t$$

$$\int v dx = \int \sin t \cdot dt = -\cos t$$

Using

$$\int u \cdot v dx = u \int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$$

$$\begin{aligned}\int t \sin t \, dt &= t(-\cos t) - \int 1 \cdot (-\cos t) \, dt \\ &= -t \cos t + \int \cos t \, dt \\ &= -t \cos t + \sin t + C.\end{aligned}$$

$$\int \sin \sqrt{2}x \, dx = -\frac{1}{\sqrt{2}} \cos \sqrt{2}x + C$$

Ex 4 // Pg 238.

$$\int e^{ax} \sin bx \, dx = ?$$

Ans

Let

$$e^{ax} = u$$

$$ae^{ax} = \frac{du}{dx}$$

$$v = \sin bx$$

$$\frac{dv}{dx} = -\frac{\cos bx}{b}$$

Using

$$\int u \cdot v \, dx = u \int v \, dx - \int \left\{ \frac{du}{dx} \int v \, dx \right\} dx$$

$$\begin{aligned}\therefore \int e^{ax} \sin bx \, dx &= e^{ax} \left(-\frac{\cos bx}{b} \right) - \int \left(ae^{ax} \cdot \left(-\frac{\cos bx}{b} \right) \right) dx \\ &= -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b} \int e^{ax} \cos bx \, dx\end{aligned}$$

consider $I = \int e^{ax} \sin bx \, dx$

$$I = -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b} \int e^{ax} \cos bx \, dx$$

$$I = -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b} \left[\frac{e^{ax} \sin bx}{b} - \int \frac{ae^{ax} \sin bx}{b} dx \right]$$

$$I = -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b^2} e^{ax} \sin bx - \frac{a^2}{b^2} \int e^{ax} \sin bx \, dx$$

$$= -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b^2} e^{ax} \sin bx - \frac{a^2}{b^2} I$$

(3)

(xii) (xiii).
Q 12, 13

$$I + I \frac{a^2}{b^2} = \frac{1}{b} e^{ax} \cos bx + \frac{a}{b^2} e^{ax} \sin bx$$

$$I \left(\frac{a^2 + b^2}{b^2} \right) = \frac{-b e^{ax} \cos bx + a e^{ax} \sin bx}{b^2}$$

$$I = \frac{e^{ax}}{a^2 + b^2} \left[-b \cos bx + a \sin bx \right]$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} \left[a \sin bx - b \cos bx \right] + C_2$$

Q 4 Calculate. ?

(i) $\int e^x (\sin x + \cos x) dx = ?$

Using

$$\int e^{ax} [a f(x) + f'(x)] dx = e^{ax} f(x) + C$$

$$\therefore \int e^x (\sin x + \cos x) dx = e^x \sin x + C \quad \text{Ans}$$

Q $\int e^x \left(\sin x + \frac{1}{\sqrt{1-x^2}} \right) dx = ?$

Using

$$\int e^x \left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2} \right] dx$$

$$= e^x \frac{1}{(1-x)} + C \quad \left(\begin{array}{l} y = \frac{1}{1-x} = (1-x)^{-1} \\ = (-1)(x-1)^{-2} \frac{d}{dx}(1-x) \\ y = \frac{(-1)(-1)}{(x-1)^2} = \frac{1}{(x-1)^2} \end{array} \right)$$

$$4(iii) \int \frac{x e^x}{(1+x)^2} dx \rightarrow ?$$

$$\Rightarrow \int \frac{x e^x}{(1+x)^2} dx$$

$$\Rightarrow \int \frac{e^x (1+x-1)}{(1+x)^2} dx$$

$$\Rightarrow \int e^x \left[\frac{1+x}{(1+x)^2} - \frac{1}{(1+x)^2} \right] dx$$

$$\Rightarrow \int e^x \left[\frac{1+x}{(1+x)^2} - \frac{(-1)}{(1+x)^2} \right] dx$$

$$\Rightarrow e^x \frac{1}{(1+x)} + C \quad \text{Using formula} \quad \int e^{ax} [af(x) + f'(x)] dx = e^{ax} f(x) + C$$

$$\Rightarrow \frac{e^x}{1+x} + C$$

$$Q 4(vi) \int \cos \left(b \ln \frac{x}{a} \right) dx ;$$

Sol

$$\frac{x}{a} = e^u$$

$$x = a e^u \quad dx = a e^u du$$

$$\begin{aligned} \cos b \ln \frac{x}{a} &= \cos b \ln(e^u) \\ &= \cos b \ln(e^u) \\ &= \cos bu \end{aligned}$$

$$\int \cos \left(b \ln \frac{x}{a} \right) dx = \cos bu a e^u du$$

$$= a \int \cos bu e^u du$$

$$\cos^5 x$$

$$\cos^4 x (\cos^2 x - 1)$$

$$a \cdot e^u \int \cos x (1 - \sin^2 x)^2 dx$$

$$\int \cos x dx - \int \sin^2 x \cos x dx$$

$$\sin x - \frac{\sin^3 x}{3} + C$$

$$\int \cos x (1 + \sin^4 x - 2\sin^2 x) dx$$

$$\int \cos x dx + \int \sin^4 x \cos x dx - 2 \int \sin^2 x \cos x dx$$

$$\rightarrow \sin x + \frac{\sin^5 x}{5} - \frac{2}{3} \sin^3 x + C$$

$$\Rightarrow \int_1^2 (x^2+x+1)^{1/3} (2x+1) dx$$

$$\Rightarrow \left[\frac{(x^2+x+1)^{4/3}}{4/3} \right]_1^2$$

$$\Rightarrow \frac{3}{4} \left[(x^2+x+1)^{4/3} \right]_1^2$$

$$\Rightarrow \frac{3}{4} \left\{ (2^2+2+1)^{4/3} - (1^2+1+1)^{4/3} \right\}$$

$$\Rightarrow \frac{3}{4} \left[7^{4/3} - 3^{4/3} \right] \quad \begin{array}{l} \sqrt[3]{7 \times 7 \times 7 \times 7} \\ 7 \sqrt[3]{7} \end{array}$$

$$\Rightarrow \frac{3}{4} \left[7 \sqrt[3]{7} - 3 \sqrt[3]{3} \right] \quad \begin{array}{l} \sqrt[3]{3 \times 3 \times 3} \\ 3 \sqrt[3]{3} \end{array}$$

$$87 \int_{-2}^1 (x^2+1)^{1/2} x dx$$

$$\Rightarrow \int_{-2}^1 (x^2+1)^{1/2} x dx$$

$$\Rightarrow \int_{-2}^1 \frac{1}{2} (x^2+1)^{1/2} 2x dx$$

$$\Rightarrow \frac{1}{3} \left[(x^2+1)^{3/2} \right]_{-2}^1$$

$$\Rightarrow \frac{1}{3} \left[(1+1)^{3/2} - (-2+1)^{3/2} \right]$$

$$\Rightarrow \frac{1}{3} [2\sqrt{2} - 5\sqrt{5}]$$

Example pg 215

$$Q \int \frac{du}{\sqrt{u+a} + \sqrt{u+b}} = ?$$

$$\Rightarrow \int \frac{du}{\sqrt{u+a} + \sqrt{u+b}} \times \frac{\sqrt{u+a} - \sqrt{u+b}}{\sqrt{u+a} - \sqrt{u+b}}$$

$$\Rightarrow \int \frac{du(\sqrt{u+a} - \sqrt{u+b})}{(u+a) - (u+b)}$$

$$\Rightarrow \int \frac{(\sqrt{u+a} - \sqrt{u+b}) du}{u+a - u - b}$$

$$\Rightarrow \frac{1}{a-b} \int (\sqrt{u+a} - \sqrt{u+b}) du$$

$$\Rightarrow \frac{1}{a-b} \left[\int \sqrt{u+a} du - \int \sqrt{u+b} du \right]$$

$$\Rightarrow \frac{1}{a-b} \left[\frac{(u+a)^{3/2}}{3/2} - \frac{(u+b)^{3/2}}{3/2} \right] + C$$

$$\Rightarrow \frac{1}{a-b} \left[\frac{2(u+a)^{3/2}}{3} - \frac{2(u+b)^{3/2}}{3} \right] + C$$

$$\Rightarrow \frac{2}{3(a-b)} \left[(u+a)^{3/2} - (u+b)^{3/2} \right] + C$$

Exmp pg 217

$$Q \int_0^1 \frac{dx}{(2x+3)^{3/2}} = ?$$

$$\int_0^1 \frac{(dx)}{(2x+3)^{3/2}}$$

$$\Rightarrow \int_0^1 (2x+3)^{-2/3} dx$$

$$\Rightarrow \frac{1}{2} \int_0^1 (2x+3)^{-2/3} \cdot 2 dx$$

$$\Rightarrow \frac{1}{2} \left[\frac{(2x+3)^{-1/3}}{-1/3} \right]_0^1$$

$$\Rightarrow \frac{3}{2} \left[(2x+3)^{1/3} \right]_0^1$$

$$\Rightarrow \frac{3}{2} \left\{ (2(1)+3)^{1/3} - (2(0)+3)^{1/3} \right\}$$

$$\Rightarrow \frac{3}{2} \left\{ \sqrt[3]{5} - \sqrt[3]{3} \right\}$$

Q Evaluate \therefore 2004

$$Q \int_0^2 (x^2+3x+5)^{2/3} \left(x+\frac{3}{2}\right) dx$$

Sol

$$\Rightarrow \frac{1}{2} \int_0^2 (x^2+3x+5)^{2/3} (2x+3) dx$$

$$\Rightarrow \frac{1}{2} \left[\frac{(x^2+3x+5)^{1/3}}{1/3} \right]_0^2$$

$$\Rightarrow \frac{3}{2} \left\{ (2^2+3 \cdot 2+5)^{1/3} - (0^2+0+5)^{1/3} \right\}$$

$$\Rightarrow \frac{3}{2} \left\{ \sqrt[3]{15} - \sqrt[3]{5} \right\}$$

Q. own

$$\int \sec x dx = \ln |\sec x + \tan x| + C$$

$$\int \frac{\sec \sqrt{x} dx}{\sqrt{x}}$$

Sol

$$\text{Let } \sqrt{x} = u$$

$$\frac{1}{2\sqrt{x}} dx = du$$

$$dx = 2\sqrt{x} du$$

$$\frac{1}{\sqrt{x}} dx = 2 du$$

$$\Rightarrow \int \sec u \cdot 2 du$$

$$\Rightarrow 2 \int \sec u du$$

$$\Rightarrow 2 \{ \ln |\sec u + \tan u| \} + C$$

Q. 1. (x iii)

$$\int (\sec^4 x - 1) dx$$

Sol

$$\Rightarrow \int (\sec^2 4x - 2 \sec 4x + 1) dx$$

$$\Rightarrow \int \sec^2 4x dx - 2 \int \sec 4x dx + \int dx$$

$$\Rightarrow \frac{\tan 4x}{4} - 2 \frac{\ln |\sec 4x + \tan 4x|}{14} + x$$

$$\int \sec^2 [3x] dx = \frac{\tan [3x]}{3} + C$$

$$\int \cot y dy = \frac{\sin [y]}{[y]'} + C$$

$$\Rightarrow \frac{\tan 4x}{4} - \frac{1}{2} (\ln(\sec 4x + \tan 4x))$$

$$\Rightarrow \frac{\tan 4x}{4} - \frac{1}{2} \ln(\sec 4x + \tan 4x) + x + c$$

$$\Rightarrow \frac{\tan 4x}{4} - \ln(\sec 4x + \tan 4x)^{\frac{1}{2}} + x + c$$

Ans

Q) $\int \frac{dx}{\csc 2x - \cot 2x}$

Sol

$$\Rightarrow \int \frac{1}{\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}} dx$$

$$\Rightarrow \int \frac{dx}{\frac{1 - \cos 2x}{\sin 2x}}$$

$$\Rightarrow \int \frac{\sin 2x}{1 - \cos 2x} dx$$

Let.

$$1 - \cos 2x = u$$

$$0 + 2 \sin 2x dx = du$$

$$\sin 2x dx = \frac{du}{2}$$

$$\Rightarrow \int \frac{1}{u} \cdot \frac{du}{2}$$

$$\Rightarrow \frac{1}{2} \int \frac{1}{u} \cdot du$$

$$\Rightarrow \frac{1}{2} \ln u + c$$

$$\Rightarrow \frac{1}{2} \ln(1 - \cos 2x) + C$$

$$\Rightarrow \ln \sqrt{1 - \cos 2x} + C$$

Ans

H.W

$$Q \int \frac{dx}{\csc 5x - \cot 5x}$$

$$\Rightarrow \frac{1}{2} \ln(2 \sin^2 x) + C$$

$$\csc 5x - \cot 5x$$

$$\Rightarrow \ln \sin^2 x + C$$

$$Q. \int \frac{dx}{\csc 9x - \cot 9x}$$

Alone

$$\sin^2 \boxed{x} = \frac{1 - \cos 2\boxed{x}}{2} + C$$

N.A. Alone

$$\sin^2 \boxed{x} \cdot \sin x = 1 - \cos^2 \boxed{x} + C$$

Alone

$$\cos^2 \boxed{x} = \frac{1 + \cos 2\boxed{x}}{2} + C$$

not Alone

$$\cos^2 \boxed{x} \cdot \cos x = 1 - \sin^2 \boxed{x} + C$$

Q. 5(i)

$$\int \sin^2 x \, dx$$

$$\Rightarrow \int \frac{(1 - \cos 2x)}{2} \, dx$$

$$\Rightarrow \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x \, dx$$

$$\text{Using } \int \cos \square dx = \frac{\sin \square}{\square'} + C$$

$$\Rightarrow \frac{1}{2} x - \frac{\sin 2x}{2} \left(\frac{1}{2} \right) + C$$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{4} + C$$

Ans

$$Q \int \cos^2 x dx$$

$$\Rightarrow \int \left(\frac{1 + \cos 2x}{2} \right) dx$$

$$\Rightarrow \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x dx$$

$$\Rightarrow \frac{x}{2} + \frac{1}{4} \sin 2x + C$$

$$Q \int \cos^2(3x+2) dx$$

$$\Rightarrow \text{Put}$$

$$3x+2 = u$$

$$3 dx = du$$

$$dx = \frac{du}{3}$$

$$\Rightarrow \int \cos^2 u \frac{du}{3}$$

$$\Rightarrow \frac{1}{3} \int \cos^2 u du$$

$$\Rightarrow \frac{1}{3} \int \left(\frac{1 + \cos 2u}{2} \right) du$$

$$\Rightarrow \frac{1}{6} \int du + \frac{1}{6} \int \cos 2u du$$

$$\Rightarrow \frac{u}{6} + \frac{1}{6} \left(\frac{\sin 2u}{2} \right) + C$$

$$\Rightarrow \frac{(3x+2)}{6} + \frac{1}{12} \sin 2(3x+2) + C$$

$$= \frac{(3x+2)}{6} + \frac{1}{12} \sin 2(3x+2) + C$$

Ans

$$\int \sin^2 x \cos^2 x \, dx$$

$$\int \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 + \cos 2x}{2} \right) dx$$

$$\frac{1}{4} \int (1 - \cos 2x)(1 + \cos 2x) \, dx$$

$$\frac{1}{4} \int (1 - \cos^2 2x) \, dx$$

$$\frac{1}{4} \int dx - \frac{1}{4} \int \cos^2 2x \, dx \quad \frac{1 + \cos 2\theta}{2} = \cos^2 \theta$$

$$\frac{x}{4} - \frac{1}{4} \int \frac{1 + \cos 4x}{2} \, dx$$

$$= \frac{x}{4} - \frac{1}{8} \int (1 + \cos 4x) \, dx$$

$$= \frac{x}{4} - \frac{1}{8} \int dx - \frac{1}{8} \int \cos 4x \, dx$$

$$= \frac{x}{4} - \frac{x}{8} - \frac{1}{8} \frac{\sin 4x}{4} + C \quad \int \cos \theta \, d\theta = \frac{\sin \theta}{\theta'} + C$$

$$= \frac{x}{4} - \frac{x}{8} - \frac{1}{32} \sin 4x + C \quad \text{Ans}$$

1914-1915

1. 1st of January

2. 1st of February

3. 1st of March

4. 1st of April

5. 1st of May

6. 1st of June

7. 1st of July

8. 1st of August

9. 1st of September

10. 1st of October

11. 1st of November

12. 1st of December

13. 1st of January

14. 1st of February

$$i \Rightarrow \int (1 - \cos^2 x) \sin x \, dx$$

$$\Rightarrow \int \sin x \, dx - \int \sin x \cos^2 x \, dx$$

$$\Rightarrow -\cos x + \int (-\sin x) \cos^2 x \, dx$$

$$\Rightarrow -\cos x + \left[\frac{\cos^3 x}{3} \right] + C$$

$$\Rightarrow \frac{1}{3} \cos^3 x - \cos x + C$$

Ans

Q $\int \cos^3 x \, dx$

Sol

$$\Rightarrow \int \cos^2 x \cos x \, dx$$

$$\Rightarrow \int (1 - \sin^2 x) \cos x \, dx$$

$$\Rightarrow \int \cos x \, dx - \int \cos x \sin^2 x \, dx$$

$$\Rightarrow \sin x - \frac{\sin^3 x}{3} + C$$

Q $\int \frac{\cos^3 y}{3} \, dy$

Sol $\int \frac{\cos^3 y}{3} \, dy$

Sol

$$\Rightarrow \frac{1}{3} \int \cos^3 y \, dy$$

$$\Rightarrow \frac{1}{3}$$

$$\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1}$$

Q5 (V) $\int \sin^2 x \cos^3 x \, dx$

~~8~~

$$\Rightarrow \int \sin^2 x \cdot \overset{\text{A.M.}}{\cos^2 x} \cdot \overset{\text{N.A.M.}}{\cos x} \cdot dx$$

$$\Rightarrow \int \sin^2 x (1 - \sin^2 x) \cos x \, dx$$

$$\Rightarrow \int \sin^2 x \cos x \, dx - \int \sin^4 x \cos x \, dx$$

$$\Rightarrow \int$$